Foundations of Common Core Math 3

Unit 2A – Modeling with Linear Functions

"I'll have the math homework."

Name: ____

WAKE COUNTY
PUBLIC SCHOOL SYSTEM

APEX HIGH SCHOOL
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Foundations of Common Core Math 3

Unit 2 Linear Modelling

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Foundations of Common Core Math 3

Unit #2A Modeling with Linear Functions

Topics in this unit:

- **Linear Equations & Inequalities**
  - graph systems of linear equations and inequalities
  - create equations from real world situations to solve problems
  - identify parallel and perpendicular lines from equations
  - solve systems of linear equations graphically and algebraically
  - determine whether solutions are reasonable for equations and inequalities
- **Sequences**
  - define an arithmetic sequence
  - use recursive and explicit formulas to generate terms
  - use sequences to model situations
- **Linear Programming**
  - graph a given a set of constraints
  - determine feasible regions and evaluate maximum and minimum conditions

By the end of this unit students will be able to:

- Use, create, and solve linear equations or linear inequalities to model or represent situations.
- Solve systems of linear equations and inequalities
- Build a mathematical model to represent a real-life arithmetic situation in order to solve for the n^{th} term.
- Determine if a sequence is arithmetic.
- Identify and use recursive and explicit forms of an arithmetic sequence.
- Interpret solutions as viable or non-viable for linear equations, inequalities and in a modeling context.
- Be able to determine maximum or minimum values for a linear model situation given the constraints.

By the end of this unit, students will be able to answer the following questions:

- How are systems of linear equations and inequalities useful in interpreting real world situations?
- How can equations for parallel and perpendicular lines be recognized?
- How can linear programming be used to maximize and minimize values in real world situations?
- How are arithmetic sequences used to model mathematical ideas and realistic situations?
- How can you use a recursive and explicit formula for an arithmetic sequence?

VOCABULARY

- **An equation** is a mathematical sentence that shows that two expressions are equivalent.
- **A linear equation** is an algebraic equation in which each term is either a constant or the product of a constant and a single variable with degree 1.
- **Slope of a linear equation** is the rate of change. The ratio of the change in "y" values compared to the change in "x" values. m = $\frac{y_2 - y_1}{x_2 - x_1}$
- **A function** is a relation from a set of inputs (x) to a set of possible outputs (y); where each input is related to exactly one output.
- **A linear function** is a function whose graph is a straight line.

  Slope Intercept Form: y = mx+b where m is the slope and b is the y-intercept.

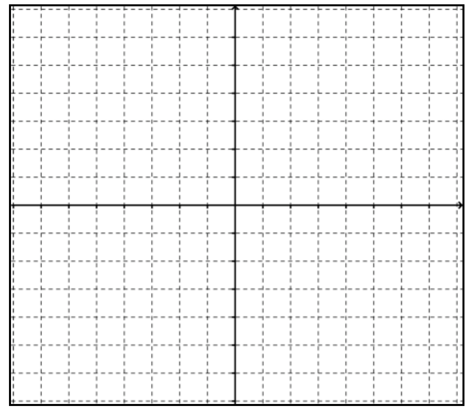
  Standard Form: Ax + By = C where A,B and C are all integers

  Point Slope Form: $y - y_1 = m(x - x_1)$ where (x_1, y_1) is any point on the line and m is the slope.
- **An inequality** is a mathematical sentence that shows that two expressions are not equivalent.
- **A system of equations** is a collection of two or more equations with the same set of variables.
- **A system of inequalities** is a collection of two or more inequalities with the same set of variables.
- **The solution to systems of equations** must satisfy all equations in the system. Two linear equations are either parallel (no solution), the same line (infinite solutions) or intersecting lines (one solution).
- **Some methods for solving a system of equations are**: graphing, substitution, and elimination
- **Parallel lines** are lines in the same plane that do not intersect; the slopes are the same, $m_1 = m_2$, and the *y* intercepts are different.
- **Perpendicular lines** are lines that intersect to form right angles; their slopes are opposite reciprocals:
- **Linear programming** is a mathematical procedure to find values for variables that satisfy a set of linear constraints and optimize the value of a linear objective function.
- **An objective function** is an algebraic expression like 7x – 3y whose value is to be either maximized or minimized within the given constraints of the problem.
- **Constraints** are limitations on values that variables may assume in a problem situation.
- **The feasibility region** is the area of the graph in which every point satisfies the constraint inequalities.
- **An arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a fixed non-zero constant.
- **A recursive rule** – A rule which uses the value of one term in the sequence to define the value of the next term in the sequence. You must state a beginning value.
- **An explicit rule** is a formula that determines any term in the sequence.

Warm-ups/Exit Tickets

Linear Equations

1. Equation: $y = -x + 3$ Graph:



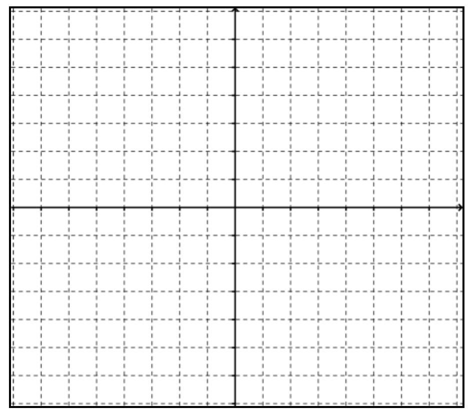
Slope: ___________

y-intercept: ___________

Table:

x	y
3	
	-3

2. Equation: $x = 2$ Graph:



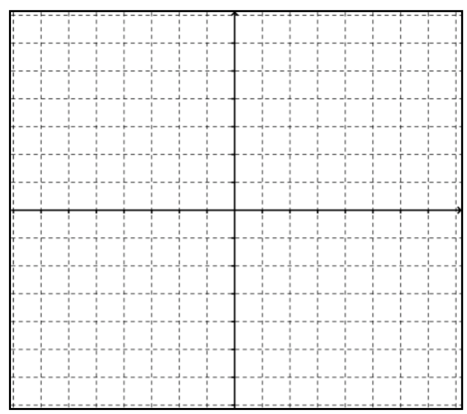
Slope: ___________

y-intercept: ___________

Table:

x	y
-4	
	5

3. Equation: $3x + 4y = 12$ Graph:



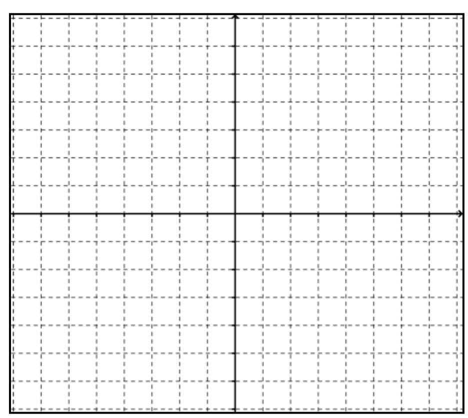
Slope: ___________

y-intercept: ___________

Table:

x	y
6	
	2

4. Equation: $y = -1$ Graph:



Slope: ___________

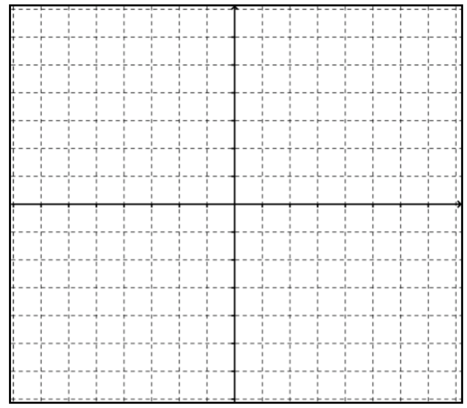
y-intercept: ___________

Table:

x	y
2	
	5

5. Inequality: $-2x + 4 > y$

Graph:

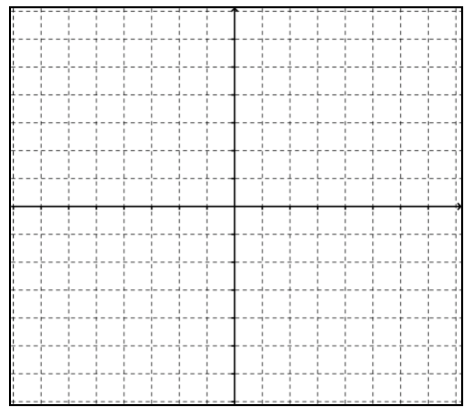


Slope: ____________

y-intercept: _____________

6. Inequality: $\frac{1}{2}x + y \leq 3$

Graph:

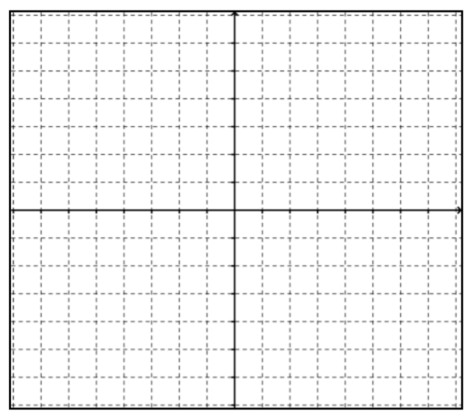


Slope: ____________

y-intercept: ____________

7. Inequality: $x - 2y \geq 4$

Graph:

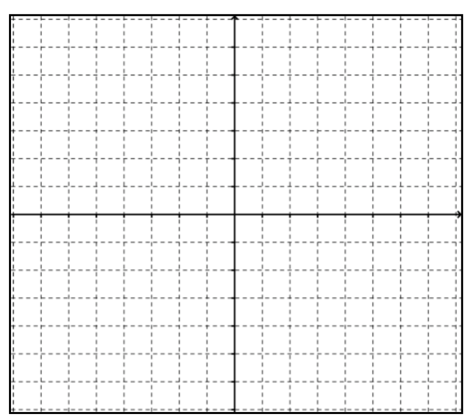


Slope: ____________

y-intercept: ____________

8. Inequality: $2y - x < -4$

Graph:



Slope: ____________

y-intercept: ____________

Use your own graph paper when needed.

Graph the line containing the given point and the given slope.

9. (1, -3) m=2 10. (0, -2) $m= -\frac{2}{3}$

11. (-1,3) undefined slope 12. (2,1) m=0

Graph using the given information.

13. y intercept is (0,-2), parallel to x + y =8

14. slope = $\frac{-2}{3}$, y intercept is the same as for the graph of y - 2x = 3

15. y intercept is (0, -1), perpendicular to y = -3x + 7

16. Parallel to the graph of 5x – y – 1 =0 and having a y intercept of (0, 3).

State whether the graphs of the following equations are parallel, perpendicular or neither.

17. $2x + y = 4$
$x - 2y = 6$

18. $\frac{1}{2}x + 2y = 1$
$x + 4y = 3$

19. $6x - 2y = 1$
$\frac{2}{3}x - y = 11$

20. $y - 5 = 0$
$3x = -9$

Graph each system of Equations

21. $3x - 2y = 6$
$6x - 4y = 12$

22. $x - 3y = 2$
$2x - 6y = 12$

23. $x=5$
$x + y = 1$

Graph each system of inequalities.

24. $y \leq 3x - 5$
$y < -\frac{1}{2}x + 4$

25. $y < x + 5$
$y \geq 4x - 2$

26. $x < -4$
$y \geq 2$

Graph the system of inequalities, and classify the figure created by the solution region.

27. $x \leq 2$
$x \geq -3$
$y \leq 2x + 2$
$y \geq 2x - 1$

28. $y \leq -x + 4$
$y \leq 3$
$y \geq 0$
$y \geq -2x - 1$

Maximize or minimize each objective function.

29. Maximize $P = 5x + 4y$

Constraints: $0 \leq y \leq 8$
$x \geq 0$
$x+y \leq 14$
$5x + y \leq 50$

30. Minimize $C = 10x + 7y$

Constraints: $0 \leq x \leq 60$
$0 \leq y \leq 45$
$5x + 6y \leq 420$

Arithmetic Sequences

31. Determine whether each is an arithmetic sequence:

a.) 4, 2, 0, -2 …   b.) 50, 60, 70, 80 …   c.) 10, 11, 13, 16, 20 …..

32. Write an explicit formula for 5, 10, 15, 20…

33. Write the first 5 terms of the sequence defined by $a_1 = 7$, $a_n = a_{n-1} - 3$

34. Find the 90th term of $a_n = 15 + (n-1)3$

35. If the 10th term is 22 and the 20th term is 52, what is the first term?

Day 1 FCC3: Functions, Linear, Slope, Graphing equations

Relation: ANY set of input/output values that can be presented as a set of ordered pairs, a graph, a mapping or a Tchart.

Function: a special relation where every input value is paired with exactly one output value.

Domain: the "input for a relation or function. The independent variable. "X"

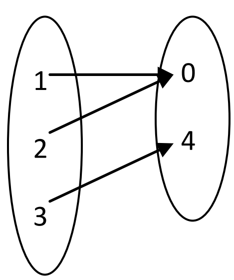
Range: the "output" value, the dependant variable, the "Y"

Passes the **Function test** if: with ordered pairs – no two timing "girls"

with graphs – no vertical line can hit more than one point

Examples: is this a function? Give domain and range.

1.)



X	Y
2	3
2	4
1	0
3	1

3.) (2,0) (2, -3)

4.) (2,3) (4,3)

Linear functions: graph to give a straight line and DO NOT contain exponents on variables, variables in the denominator or the product of variables. There are 3 different forms of linear equations we will be working with:

Slope intercept form: $y = mx+b$

Point slope form: $y - y_1 = m(x - x_1)$

Standard form: $Ax + by = C$ where A, B and C are integers and A is positive.

Example: is it linear? If so, name domain and range.

1.) $2x - 3y = 4$

2.) $x^2 + y^2 = 1$

3.) $x = -1$

4.) $y = \frac{2}{3}x - 4$

Slope: is defined to be "m" for slope intercept and point slope equations, $\frac{rise}{run}$ from a graph, $\frac{y2-y1}{x2-x1}$ from two ordered pairs, and $\frac{-A}{B}$ from standard form. A **horizontal** line has a "0" slope and a **vertical** line does not have slope, and therefore its slope is "undefined" or $\emptyset$. **Parallel** lines have the same slope and **perpendicular** lines have opposite reciprocal slopes.

Examples: determine the slope for each situation:

1.) X = 5

2.) 3x – 4y = 8

3.) y – 2 = 3 (x + 1)

4.) (4, -2) (-3, 6)

5.) (-4, 2) (-4, 1)

6.) (7, 5) (-4, 5)

Graphing a linear equation: find one point from the equation to plot first, then use slope to get the second point, then draw the line.

Examples: USE SEPARATE GRAPH PAPER for ALL the EQUATIONS

1.) y = -3x + 4 first point is the y intercept (0,4)
Slope is $\frac{-3}{1}$ which means down 3 and 1 to the right

2.) y-3 = $\frac{-1}{2}$ x(x + 2) first point is (-2, 3) and slope is down 1 and right 2

3.) 4x – 2y = 2 first point can be the x intercept ($\frac{1}{2}$, 0) or the y intercept (0,-1)
The slope is $\frac{-A}{B}$ which for this equation is $\frac{-4}{-2}$ = 2 so up 2 and right 1

4.) x = -3 first point is (-3, anything) and slope is undefined

5.) y = 2 first point is (anything, 2) and slope is 0

6.) graph the line having a slope of -2 with the same y – intercept as 3x + 2y = 6

7.) graph the line parallel to x – 4y = 4 and passing through (-2, 0)

8.) graph the line perpendicular to y = $\frac{2}{3}$ x + 5 and containing the same y intercept as y = 2

9.) graph the line passing through the x – intercept of 3x – y = -9 and the y – intercept of x + 2y = -6

10.) graph the line parallel to 2y - 6 = 10 containing the point (-3,1)

Selling Credit Cards: Companies that offer credit cards pay the people who collect applications for those cards and the people who contact current cardholders to sell them additional services.

1. For collecting credit card applications, Barry's daily pay B is related to the number of applications he collects n by the rule B = 5n + 20.

a. Use the rule to complete this table of sample (n, B) values:

Number of Applications	0	1	2	3	4	5	10	20	25
Daily Pay (in dollars)									

b. Use the words *NOW* and *NEXT* to write a rule showing how Barry's daily pay changes with each new credit card application he collects.

c. Let's draw a graph of the above table:

Label each axis.

2. The table shows how much Cheri earns for selling selected numbers of additional services.

Number of Services old	0	10	15	20	25	30	40	50	100
Daily Earnings (in dollars)		60		80		100	120	140	

a. Does Cheri's daily pay appear to be linear? Explain.

b. Calculate the missing entries in the table above.

c. Using the table, determine the rate of change in Cheri's daily pay as the number of services she sells increases.

d. Use the words *NOW* and *NEXT* to write a rule showing how Cheri's pay changes with each new additional service she sells.

e. Consider the following rules. C = 2 + 40n C = n + 2 C = 40 + 2n C = 50 + 1/2 n C = 2n + 50

Which of the rules show how to calculate Cheri's daily pay C for any number of services n she sells?

How do you know?

Day 2: Linear Applications Classwork(evens) and Homework (odds)

1.) Consider the model $y = \frac{4}{9}x + \frac{7}{3}$ for the relationship between price and quality of chewing gum (x = cost and y = rating).

a.) Explain the meaning of slope in the context of the problem.
b.) Explain the meaning of the y-intercept.
c.) Use the model to predict the cost of gum with a rating of 8.
d.) Use the model to predict a rating for gum costing 4 cents per stick.

2.) The size of a book is determined by the thickness of the covers and the number of pages. If the front and back covers are each 3 mm. thick, and every 25 pages adds one mm:
a.) Write a linear equation to represent the situation.
b.) How thick would a 500 page book be?
c,) If a book cannot exceed 48 mm. in thickness, what is the maximum number of pages it could have?
d.) Interpret the meaning of the slope.
e.) Interpret the meaning of the y intercept.

3.) A train averages 80 miles per hour (mi/h), and a bus averages 50mi/h. Let x be the number of hours you ride the train and let y be the number of hours you ride the bus.
a,) Write an expression for the total number of miles you ride.
b.)How far do you go if you ride:
i. 3 hours on the train and 6 hours on the bus:
ii. 6 hours on the train and 3 hours on the bus?
c.)You make a 400-mile trip, part by bus. Write an equation stating this fact.
d.)Find the intercepts of the equation and use them to plot the graph of the equation.
e.)If you go for 2 hours on the train, how many hours must you ride the bus to complete the 400-mile trip? Show that this ordered pair is on your graph in part (d).

4.) Ida Clare walks at a rate of 100 yards per minute and rides her bike at 300 yards per minute. Let x be the number of minutes she walks and let y be the number of minutes she rides.
a,)Write an expression for the total number of yards she goes.
b.) How far does she go if she:
i. walks for 7 minutes and rides for 3 minutes:
ii. walks for 3 minutes and rides for 7 minutes?
c.)Ida must deliver a package to a destination 6000 yards away. Write an equation for this.
d.)Find the intercepts for the equation in part (c) and use these to graph the equation.
e.)If Ida walks for 12 minutes on her 6000-yard delivery, how many minutes must she ride her bike? Show this ordered pair is on your graph in part (d).

5.) A health club charges a yearly membership fee of $95 and members must pay $2.50 per hour to use its facilities. How many hours did Paul use the club last year if his bill was $515?

6.) A video club has a $15 membership fee and members can rent videos for $3 each. How many videos did Marcy rent if her total bill was $$66?

7.) In 1999, there were 1,984,000 males enrolled in a 4 yr. public college. The number has increased an average of 25,000 per year since.

a.) Model this with a linear equation.

b.) Use the equation to estimate the number of males enrolled in 2010.

8.) In 1990 the life expectancy for a male born in the USA was 71.8 years. This has been increasing at the rate of 0.1 years every year.

a.) Model this with a linear equation.

b.) Use the equation to predict the life expectancy for a male born in 2000.

9.) According to the Neilson Media Research in 2000, there were 68.5 million cable TV users – which was up from the 64.7 million in 1996.

a.)Find a linear equation to model this situation.

b.)Use the equation to estimate when 100 million households will be using cable TV.

10.)Mike intends to buy a car. He must pay the price of the car plus $35 for tax/title, 6% sales tax, 1.2% uninsured motor fund, and $45 automobile club fee.

a.) Write a linear equation that can be used to compute the total cost of the car.

b.) If the base price of the car is $3500 – what will be his total cost?

c.) If the final cost of the car was $10,900 what was the base cost?

11.)In 1992 there were 2.0 hospice patients for every 10,000 people in the US. In 1994 there were 2.3, in 1996 there were 2.2 and in 1998 there were 3.0.

a.) What is the average yearly increase from 1992 to 1998? (HINT: that means slope!)

b.) Use your slope and the 1992 data point to find a linear equation to represent the problem.

c.) Predict how many people (per 10,000) are expected to use hospice in 2010.

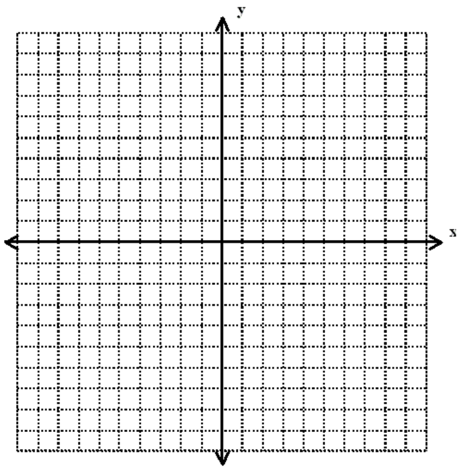
Day 3: More Linear Equation Practice

Slope-intercept form: $y = mx + b$

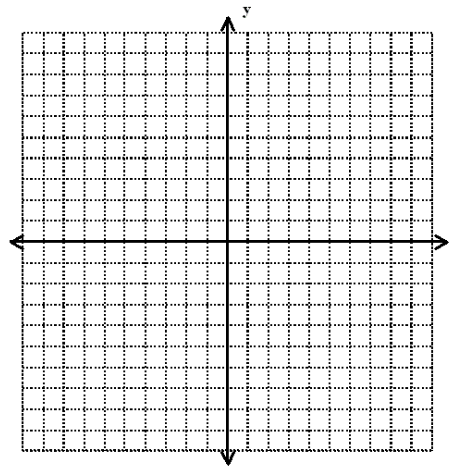
How do I graph equations in slope-intercept form?

$m = \frac{rise}{run}$ b = y-intercept

$y = \frac{1}{2}x + 7$



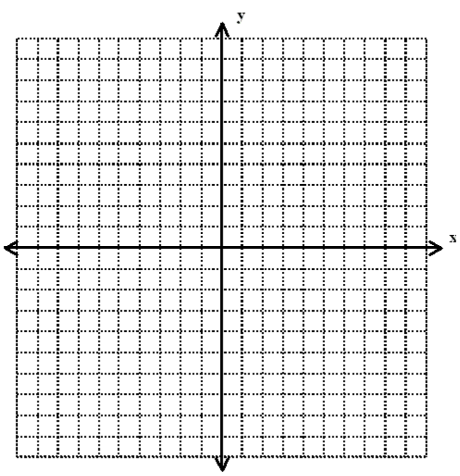
$y = -2x + 1$



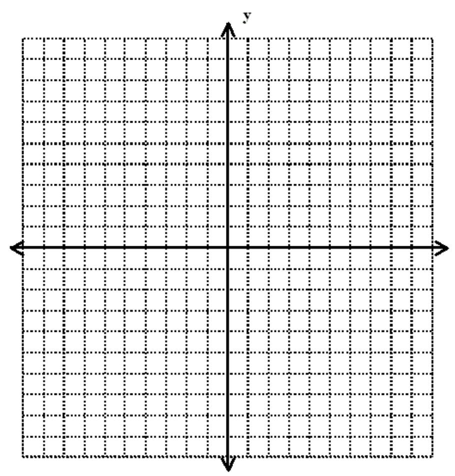
How do I graph equations in standard form?

"Cover up" to find intercepts. OR Solve for y to get into slope-intercept form.

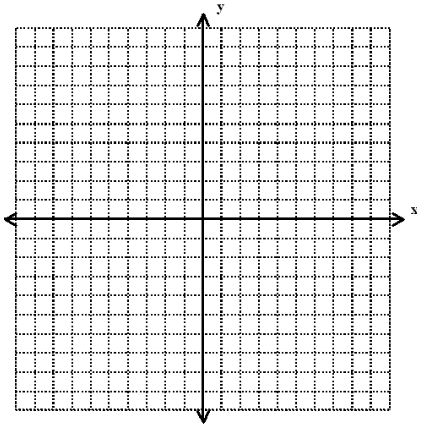
$3x + 4y = 12$



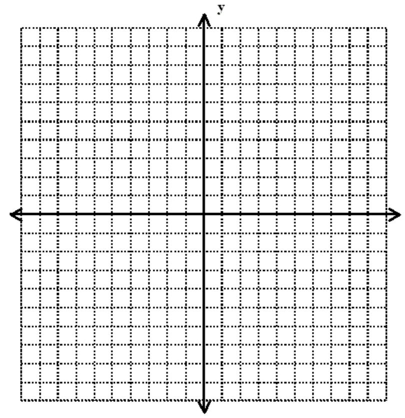
$-2x - 3y = 9$



$10x + 2y = 8$

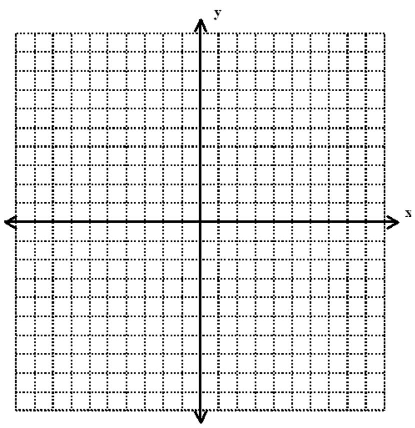


$2x - y = 6$

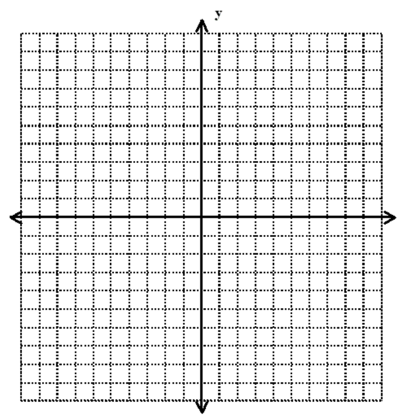


Sometimes you may have to change the scale of your graph……….

Sarah is paid a wage of $500 per week at her job. She earns $15 for each hour of overtime. Write an equation and graph.

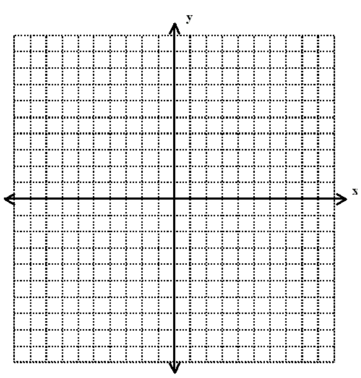


A 100 point test has x questions worth 2 points apiece and y questions worth 4 points apiece. Write an equation and graph.

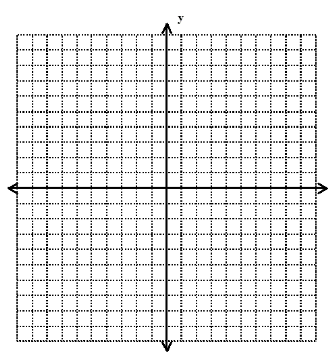


Graph each of the following. Use intervals of 1 for each axis if you can. Make sure to label your intervals.

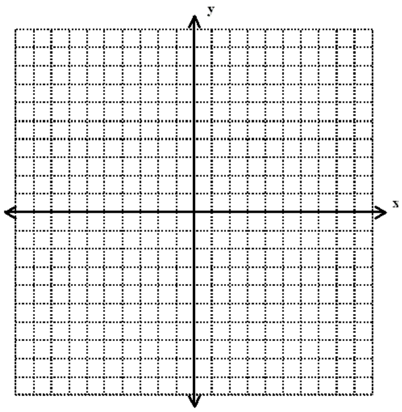
1. $y = -\frac{1}{2}x + 5$



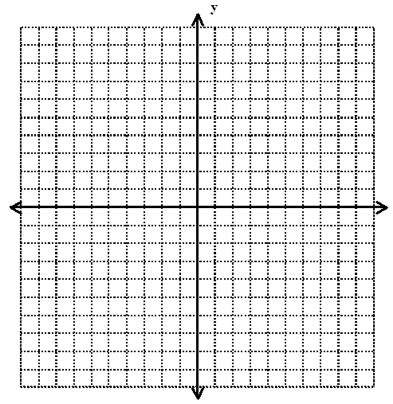
2. $y = -\frac{3}{4}x + 1$



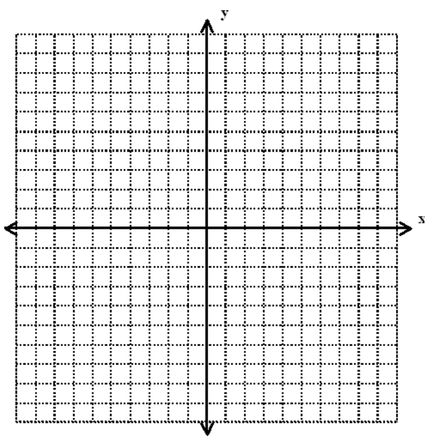
3. $3x + 2y = 8$



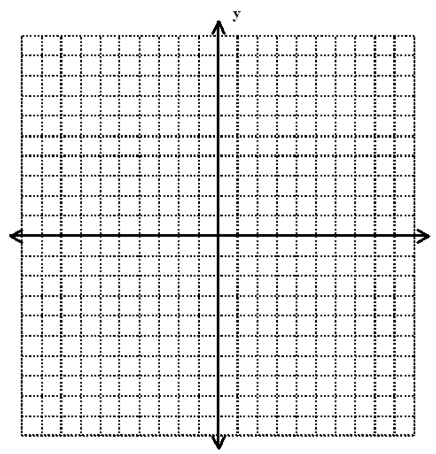
4. $x + 5y = -10$



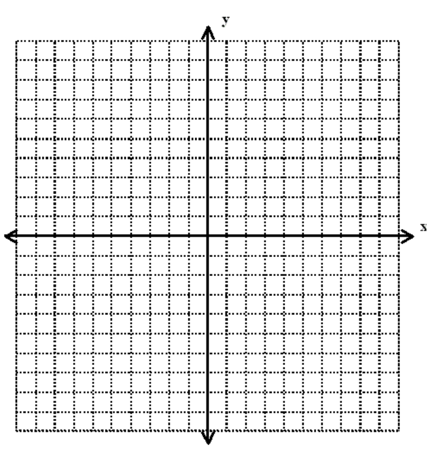
5. $7x - 3y = -15$



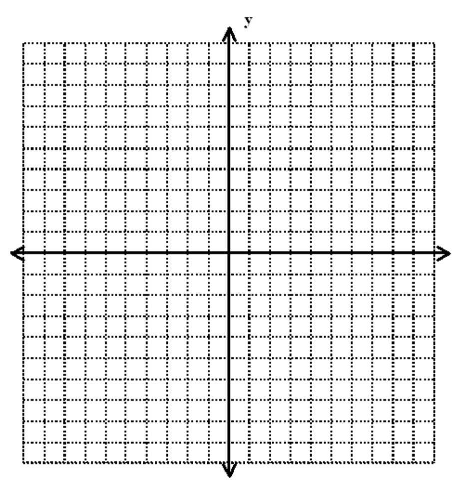
6. $y = 3x - 2$



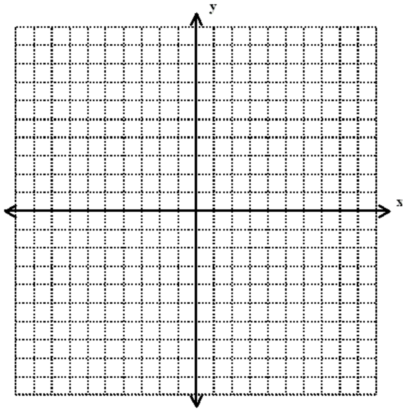
7. $y = -6x$



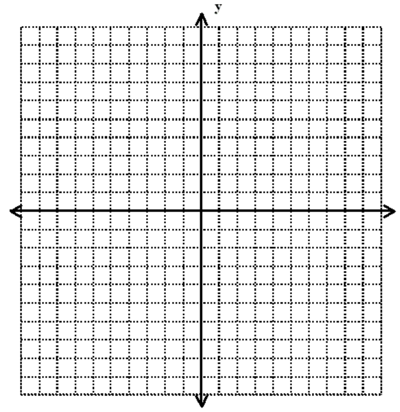
8. $y = -x + 4.5$



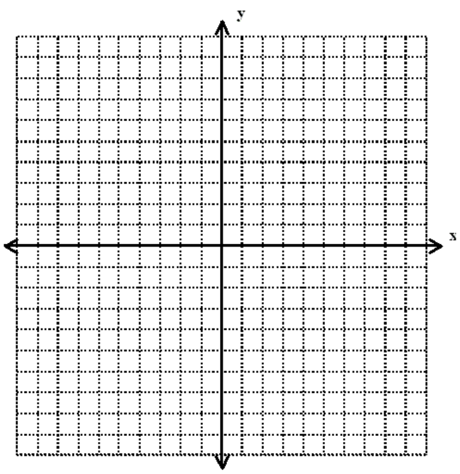
9. $y = 11x + 100$



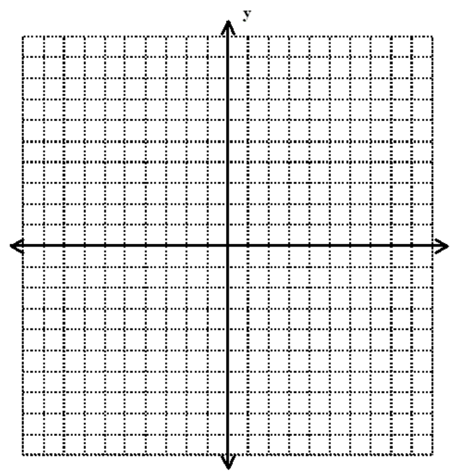
10. $y = 25x - 150$



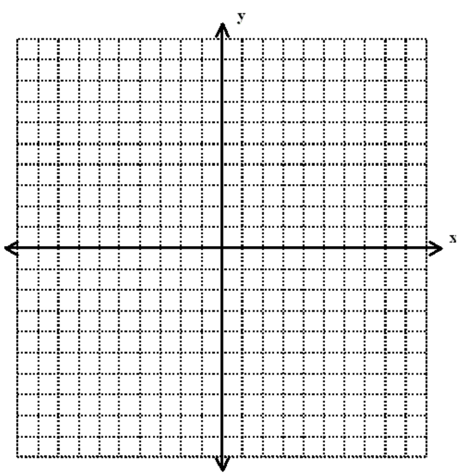
11. y + 3 = 0



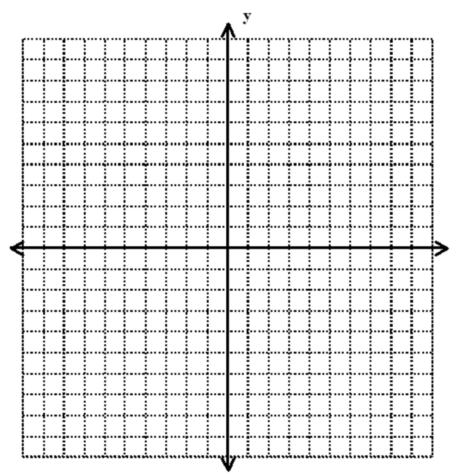
12. 5x – 20 = 10



13. $12x - 15y = 60$



14. $-8x + 10y = 120$



USE A SEPARATE SHEET OF GRAPH PAPER FOR PROBLEMS 17-24. Find the initial value and rate of change, then write an equation that represents the problem in y=mx+b form. Graph your equation on a separate sheet of graph paper, <u>clearly labeling your intervals and what each axis represents</u>. (*Adapted from Accelerated Algebra 1, Section 8.5*)

17. You are visiting Baltimore, MD and a taxi company charges a flat fee of $3.00 for using the taxi and $0.75 per mile.

18. A plumber charges $50 to make a house call. He also charges $25 per hour for labor.

19. An airplane 30,000 feet above the ground begins descending at the rate of 2000 feet per minute. Assume the plane continues at the same rate of descent. The plane's height and minutes above the ground are related to each other.

20. Lynn is tracking the progress of her plant's growth. Today the plant is 5 cm high. The plant grows 1.5 cm per day.

21. A plane loses altitude at the rate of 5 meters per second. It begins with an altitude of 8500 meters. The plane's altitide is a function of the number of seconds that pass.

Write an equation in standard form. Then use the intercepts or solve for y to graph the problem. <u>Clearly label your intervals and what each axis represents.</u> (*Adapted from braingenie.ck12.org*)

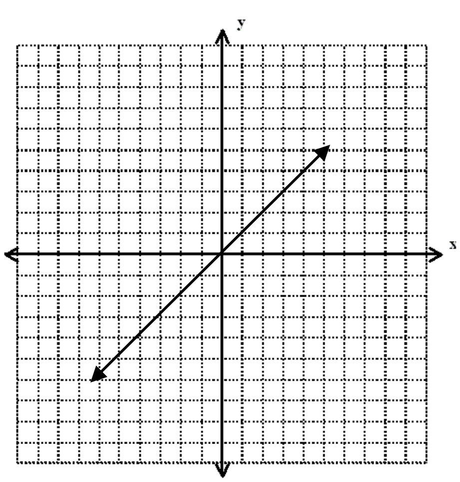
22. Louise has $36 in five-dollar bills and singles. Let x be five-dollar bills and y be singles.

23. The Ramy family bought 4 sandwiches and 3 salads. They spent $24. Let x be the cost of a sandwich and y be the cost of a salad.

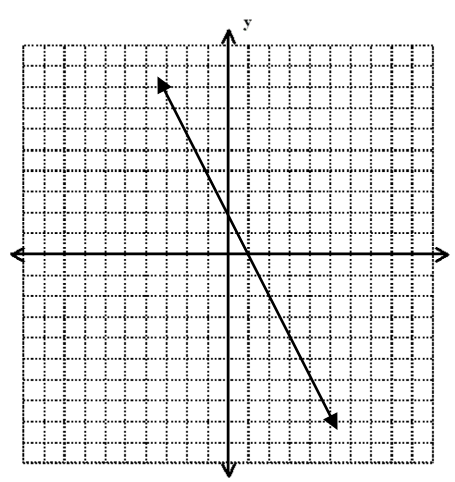
24. It will take 20 points to make the playoffs, the hockey team coach told the players. "We get 2 points for a win and one point for a tie." Let x be the number of wins and y be the number of ties.

Find the slope-intercept form of the equation for each graph. Then write a story that fits the situation.

25.



26.



Day 3 Continued: Graphing Inequalities Notes and Practice

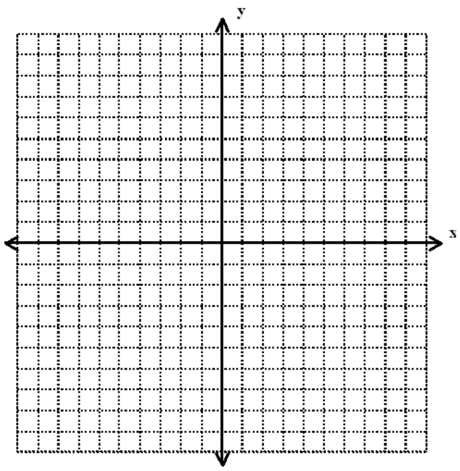
What's different about graphing inequalities?

When are solutions on the line? When are solutions not on the line?

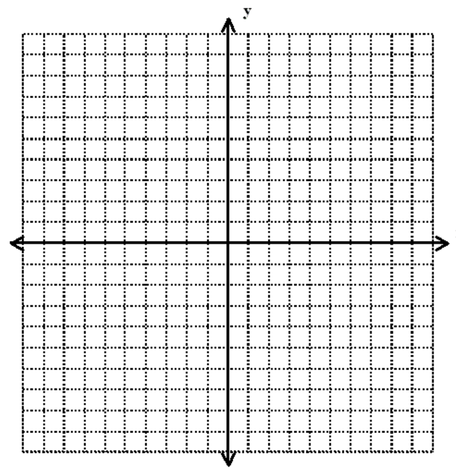
How do you decide when the line is solid or dotted?

How do you decide where to shade?

1.) $y > 2x + 2$



2.) $y \leq 3x + 1$



For which example can you write 2 solutions on the line? Name them.

For each example, name 5 solutions not on the line.

How do I write a linear inequality from a story problem?

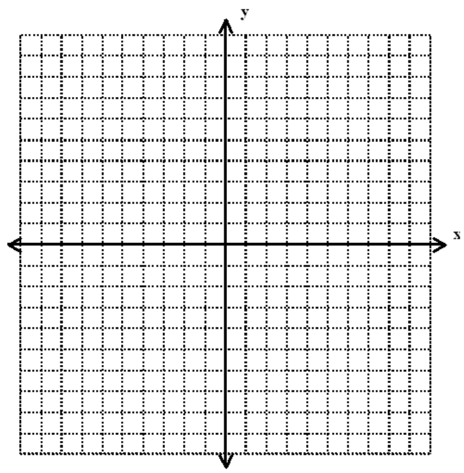
Define your variables

Look for keywords to tell you what symbol to use: (list some for reference!)

Write an inequality

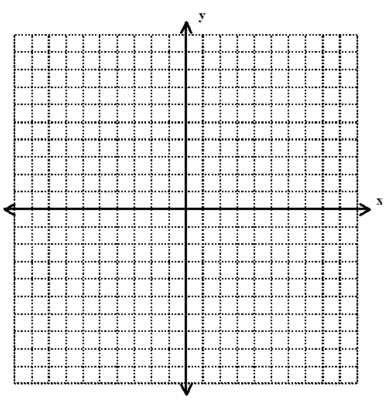
Use common sense to see if your symbol makes sense

Seth is ordering balloons for his friend’s birthday party. He has **<u>up to</u>** $15 to spend. Decorative balloons cost $3 each and solid color balloons cost $0.50 each. Write an inequality to represent how many balloons Seth can buy. Then graph the inequality.

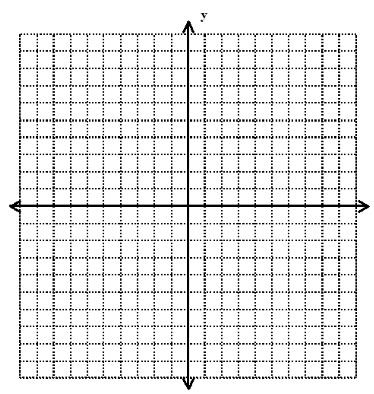


Graph each of the following inequalities.

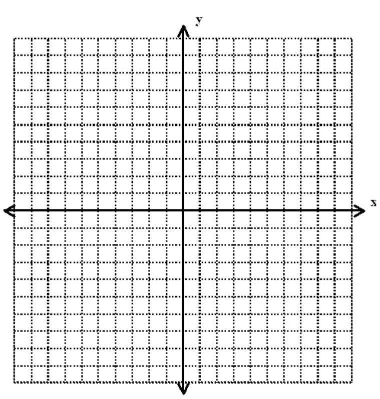
1. $y \geq 3x - 2$



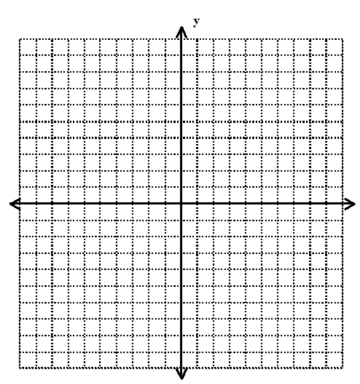
2. $y < -\frac{1}{3} + 5$



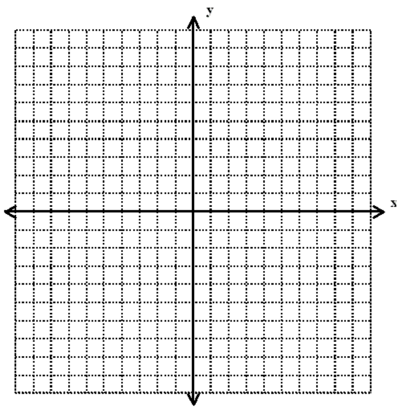
3. $y > -7x$



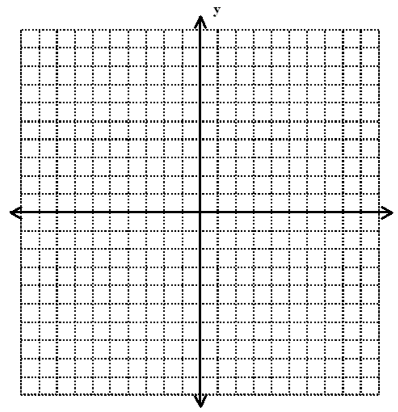
4. $y \geq -x + 4.5$



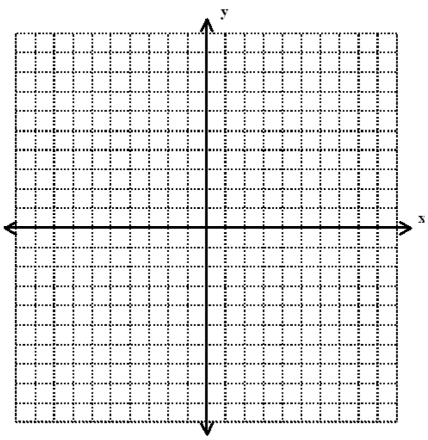
5. $y < 6x + 42$



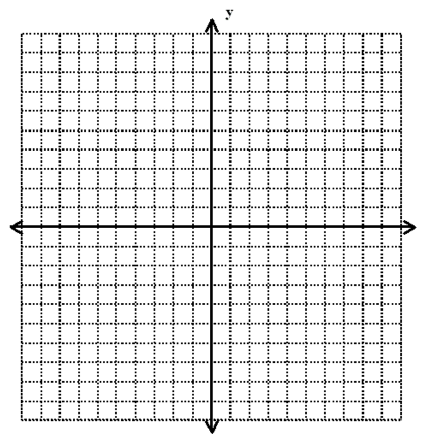
6. $y \leq 25x - 225$



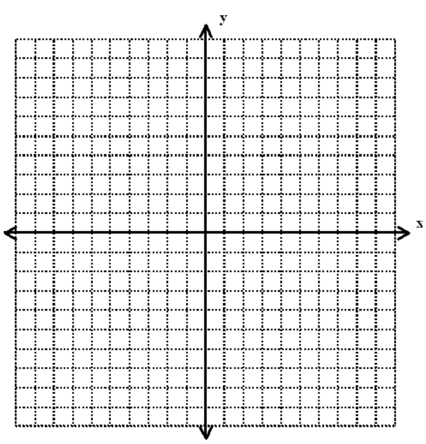
7. $2x - y > 0$



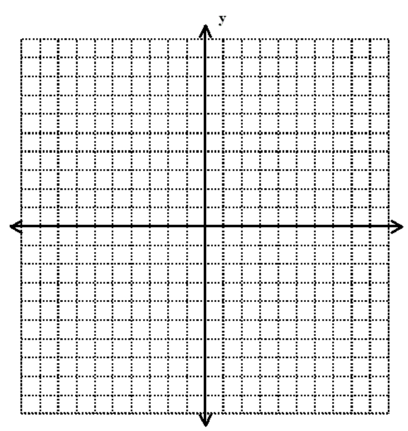
8. $x - 3y \leq -6$



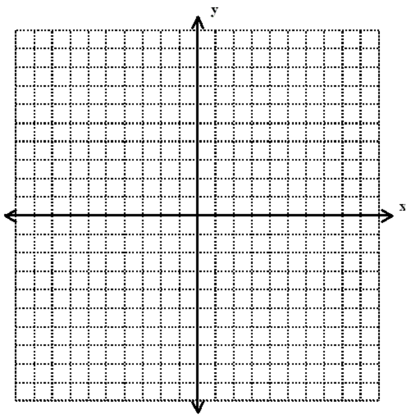
9. $6x - y \geq 4$



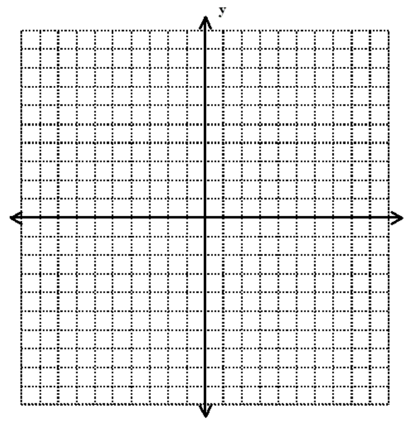
10. $x + 4y > 16$



11. $3x + 5y < 50$



12. $8x - 11y \geq -90$



For graphs 9 - 12, write 2 solutions that are on the line (if possible) and 2 solutions not on the line.

Graph #9.

Graph #10.

Graph #11.

Graph #12.

Draw a graph for each of the following story problems on a separate sheet of graph paper. Give an example of one reasonable solution and one unreasonable solution.

19. A scouting troop from Seaside is organizing a crab feed to raise money for camp. They need to make at least $910 to cover the costs of the camp. Tickets for crab dinner sell for $22 apiece. Those people who don't like crab can purchase the vegetarian dinner ticket for $20 each.

20. Susan has up to 457 minutes to dedicate to working out this week. It takes her 33 minutes to complete her cardio workout and 39 minutes to complete her weightlifting workout.

21. Wesley is selling cups of lemonade at a stand outside his house. He has enough supplies on hand to make a maximum of 289 ounces of lemonade. A regular cup holds 11 ounces of lemonade and a small cup can hold 7 ounces of lemonade.

22. Eve is going to purchase some writing instruments at the school store, where mechanical pencils cost $1 and pens cost $3. She can spend up to $13 but not more.

23. Angie is sewing some clothing. She has 98 yards of fabric to use. She uses 6 yards of fabric to make a simple dress and 18 yards of fabric to make a fancier dress with draping layers.

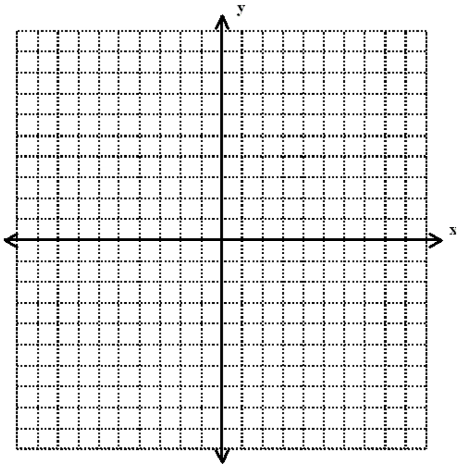
Day 4: Solving Systems of Equations and Inequalities by Graphing

What is a system of equations? How do I find the solution?

$y = \frac{1}{3}x + 3$

$2x - 4y = -8$

Graph each equation.



Find 2 solutions for the first line.

Find 2 solutions for the second line.

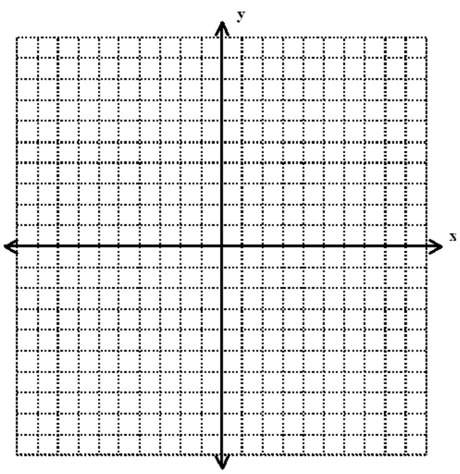
Is there a point that is a solution for BOTH lines?

What is a system of inequalities? How do I find the solutions?

$y \leq 4x + 5$

$-3x + 4y > 6$

Graph the inequalities.



Find 2 solutions for the first inequality.

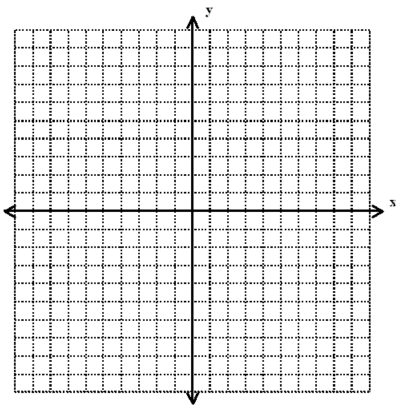
Find 2 solutions for the second inequality.

Find 2 solutions that work for BOTH inequalities.

Find the solution to each system of equations by graphing.

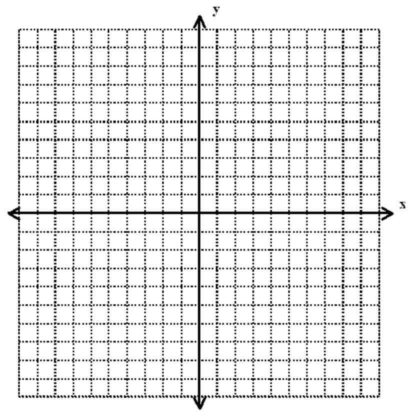
1. $y = \frac{3}{2}x - 3$

   $y = \frac{1}{4}x + 2$



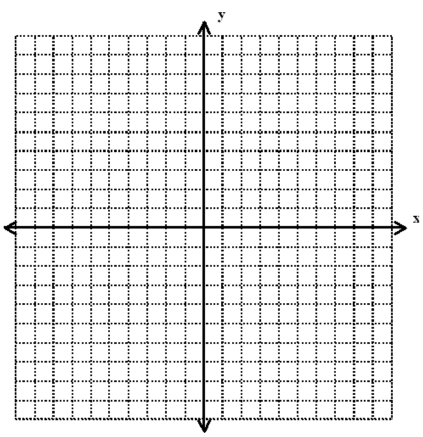
2. 2x – 4y = 16

   $x - 2y = 8$



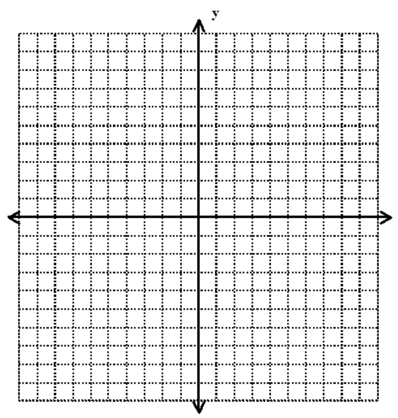
3. $x + 2y = -6$

   $7x + 2y = 6$



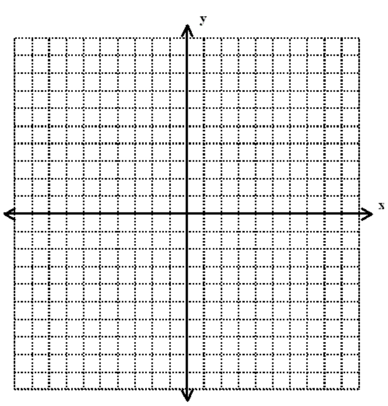
4. $y = \frac{2}{3}x + 4$

   $y = \frac{2}{3}x - 4$



Draw two intersecting lines. Write the equation of the lines. Then write the solution to the system.

5.



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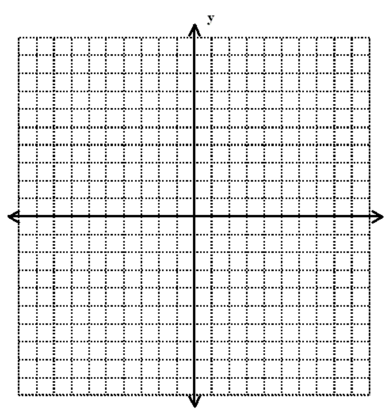
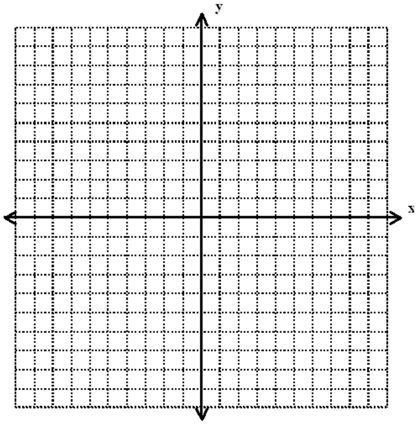
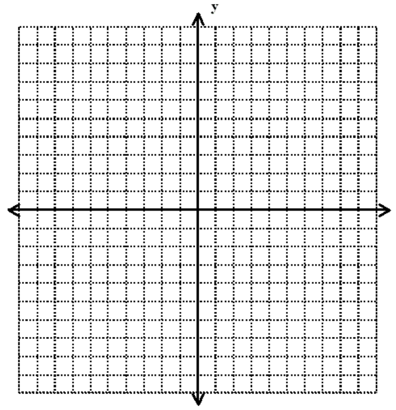


Find 3 specific solutions for each system by graphing the inequalities. Find one specific non-solution for each system, show algebraically why that solution does NOT work.

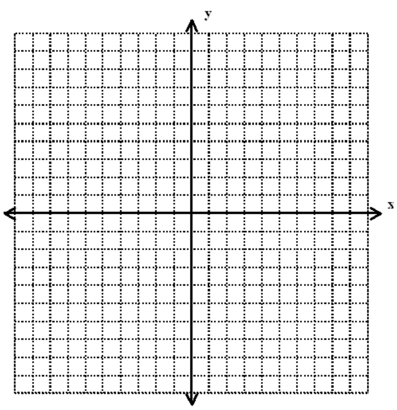
7. $y > 2x + 1$
   $y \le -x - 2$



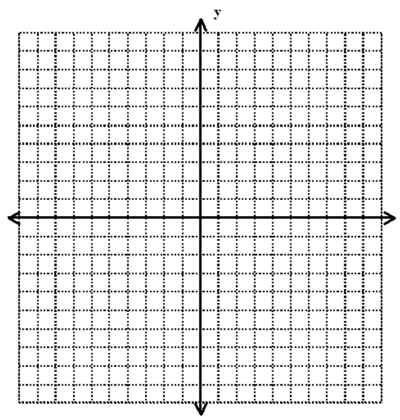
8. $y > -\frac{7}{4}x - 3$
   $x \ge 4$



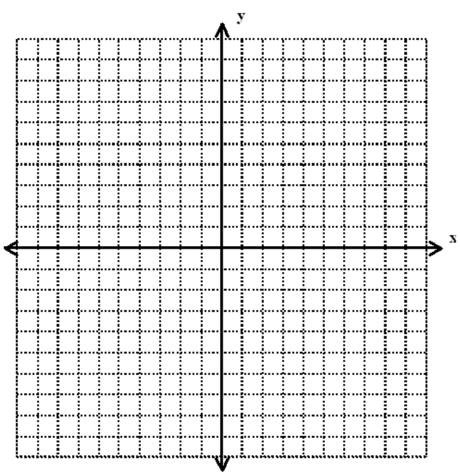
9. $x - y < 2$
   $5x + y > -4$



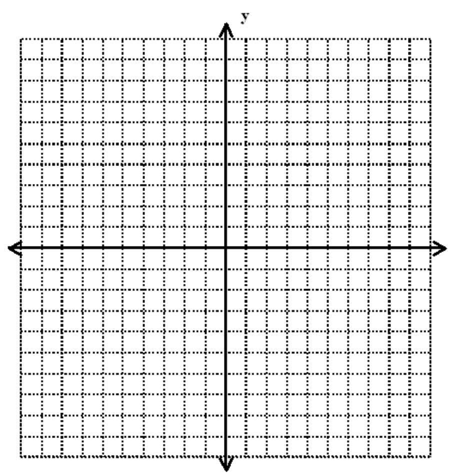
10. $x - 4y \le -12$
    $2x - y > 4$



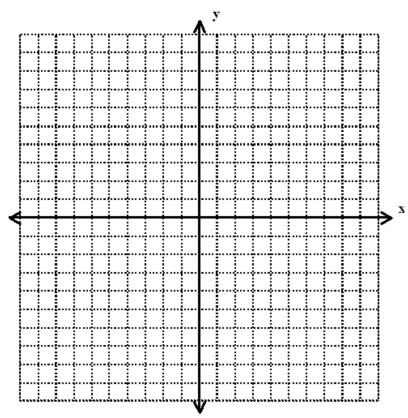
11. $y > 5$
    $x \le -7$



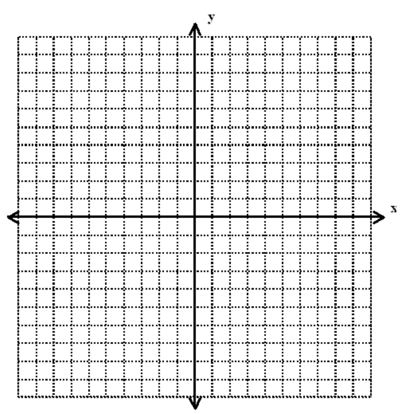
12. $y < -1$
    $x > 2$



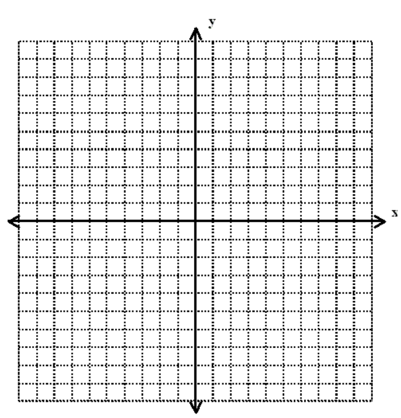
13. **Draw a system of 2 equations that has a single solution of (5, -1). Then write the equations.**



14. **Draw a system of 2 inequalities that has infinitely many solutions including (3, -7) Then write the inequalities.**



15. **Draw a system of 2 inequalities that has no solution. Then write the inequalities.**



Day 5: Linear Programming Notes and Practice

Linear Programming Vocabulary

linear programming   objective function   constraints   feasibility region   vertices

EXAMPLE: A carpenter makes bookcases in 2 sizes, large and small. It takes 4 hours to make a large bookcase and 2 hours to make a small one. The profit on a large bookcase is $35 and on a small bookcase is $20. The carpenter can only spend 32 hours per week making bookcases and must make at least 2 of the large and at least 4 of the small each week. How many small and large bookcases should the carpenter make to maximize his profit? What is the maximum profit you can find?

a.) How long would it take him to make 10 small and 5 large bookcases? Does he have enough time?

b.) How long would it take to make 5 small and 4 large? Does he have enough time? What would his profit be?

c.) Could he make 6 small and 5 large? What profit would that be?

d.) Can e make 10 small and 3 large? Profit?

e.) SO……………if there are many solutions, how do we find the one that maximizes profit???

Define your variables: X= number of small bookcases to make

Y= number of large bookcases to make

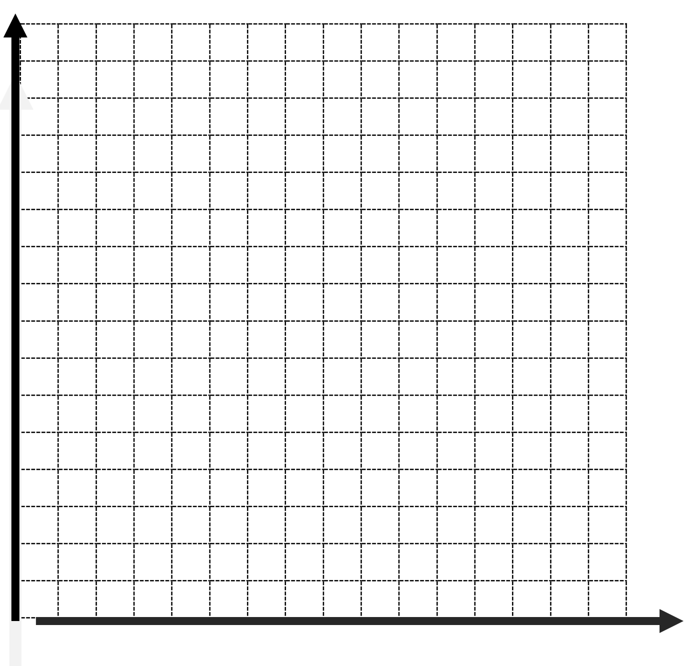
Write inequalities (CONSTRAINTS):

$2x + 4y \leq 32$ (time constraint)

$0 \leq y \leq 2$ (large bookshelf constraint)

$0 \leq x \leq 4$ (small bookcase constraint)

Write the objective function: Profit = $20x + 35y$



Graph the system of inequalities:

Highlight the shaded region (the FEASIBILITY REGION)

Label the vertices of that region.

Plug each vertex into the objective function.

Choose the one that gives the best anwer.

Use your own graph paper for problems 1-6:

1. p = x + 5y, find the maximum profit under these constraints:

$$\begin{cases} x+y \le 5 \\ x+2y \le 8 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

2. p = 4x – y, find the maximum profit under these constraints. (ans:$20)

$$\begin{cases} x+y \le 6 \\ 2x+y \le 10 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

3. c = 2x + 2y, find the minimum costs under these constraints:

$$\begin{cases} 2x+y \le 6 \\ x \ge 0 \\ y \ge 2 \end{cases}$$

4. c = x + 3y, find the minimum costs under these constraints: (ans: $2)

$$\begin{cases} x+2y \le 8 \\ x \ge 2 \\ y \ge 0 \end{cases}$$

5. If profit is represented by p = 3x + 4y, find the maximum profit under these constraints:

$$\begin{cases} x+y \le 3 \\ x \ge 0 \\ y \le 2 \end{cases}$$

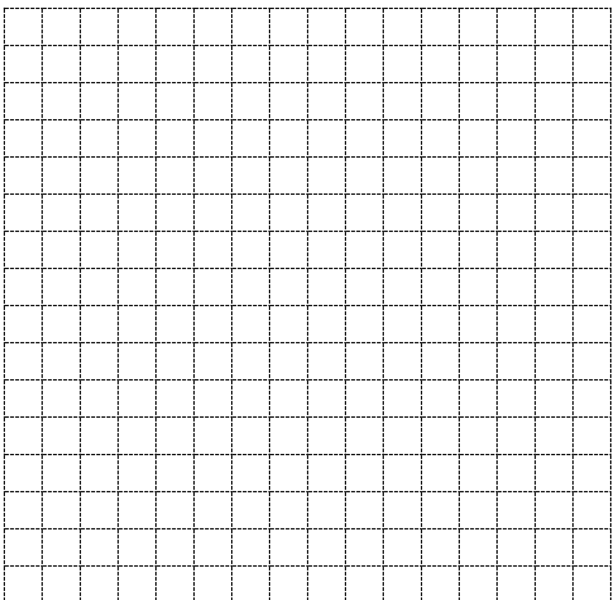
6. If cost is represented by c = 2x + 3y, find the minimum costs under these constraints: (ans: $7)

$$\begin{cases} x+y \le 5 \\ x \ge 2 \\ y \ge 1 \end{cases}$$

7. A ski company makes two types of skis and has a fabrication and a finishing department. A pair of downhill skis requires 6 hours to fabricate and 1 hour to finish. A pair of cross-country skis requires 4 hours to fabricate and 1 hour to finish. The fabricating department has 108 hours of labor available per day. The finishing department has 24 hours of labor available per day. The company makes a profit of $40 on each pair of downhill skis and $30 on each pair of cross-country skis. How many of each type should the manufacturer produce to maximize his profit? What is the maximum profit that you can find?

Define your variables: X= Y=

Write inequalities: Write the objective function:



Graph the system:

Shade the feasibility region and label the vertices.

Test points and find the answer that maximizes the objective function.

Answer??

8. The B & W Leather Company wants to add handmade belts and wallets to its product line. Each belt nets the company $18 in profit, and each wallet nets $12. Both belts and wallets require cutting and sewing. Belts require 2 hours of cutting time and 6 hours sewing time. Wallets require 3 hours of cutting time and 3 hours of sewing time. If the cutting machine is available 12 hours a week and the sewing machine is available 18 hours per week, what combinations of belts and wallets produces the greatest profit?

Define your variables: X= Y=

write inequalities: Write the objective function:

Graph the system and shade the feasibility region

Label the vertices

Test them in the objective function

Answer??

ON A SEPARATE SHEET OF PAPER:

9. Kay grows and sells tomatoes and green beans. It costs \$1 to grow a bushel of tomatos and it takes 1 square yard of land. It costs \$3 to grow a bushel of beans and 6 square yards of land. Kay’s budget is \$15 and she has 24 sq. yd. of land available. If she makes \$1 profit on each bushel of tomatoes and \$4 profit on each bushel of beans, how many bushels of each should she grow in order to maximize profits? (ans: 6 tomatoes and 3 bean)

Day 6: Solving Systems of Equations Algebraically

Elimination

You need to add the equations together with the goal being to "eliminate" one variable.

$2x + 3y = 5$
$x - 5y = 9$

It will be easier to eliminate the "x" if you multiply the second equation by "-2"

$2x + 3y = 5$
$-2x + 10y = -18$

Now when you add the two equations, the result is: $13y = -13$, so $y = -1$

Plug your first answer back into either equation, and get the second answer.
$2x + 3(-1) = 5$
$2x - 3 = 5$
$2x = 8$
$x=4$ So the solution to the system is (4, -2) which means the system represented two intersecting lines.

Practice: Use elimination to solve each system. Show all work!

1.) $-2x + y = 6$
$4x - 2y = 5$

2.) $-x + 2y = 3$
$4x - 5y = -3$

3.) $4x + 6y = 15$
$-x + 2y = 5$

4.) $6x - y = -2$
$-18x + 3y = 4$

5.) $7x + 2y = -3$
$-14x - 4y = 6$

6.) $x - 4y = -2$
$-3x + 8y = -1$

Substitution:

The goal will be to “substitute” a “new name” for a variable into the other equation, and solve.

$2x + 3y = 5$
$x – 5y = 9$

Choose the variable with a coefficient of one (if possibole!) to get by itself. So isolate the “x” in the second equation.

$x = 5y + 9$ So “$5y + 9$” is the “new name” for “x” and can be used to replace it in the first equation since they are equivalent. So substitute it and solve for “y”.

$2(5y + 9) + 3y = 5$

$10y + 18 + 3y = 5$

$13y = -13$

$y = -1$ This is your first answer. The easiest place to plug it in to get the second answer is the “new name”

$5(-1) + 9 = 4$ so $x = 4$ The solution is $(4, -1)$ and the lines were intersecting.

Practice: Solve using substitution. Show all work!

1.) $y = 2x + 6$
$4x – 2y = 5$

2.) $x - y = 3$
$-2x + 2y = -6$

3.) $-x + 2y = 3$
$4x – 5y = -3$

4.) $6x – y = -2$
$-18x + 3y = 4$

5.) $-5x + 2y = -10$
$3x – 6y = -18$

6.) $7x + 2y = -3$
$-14x – 4y = 6$

Day 7: Arithmetic Sequence Notes

A ***sequence*** is an ordered arrangement of numbers, figures, or objects.

Recursion is the method of describing a step in a sequential process in terms of previous steps. ***Iteration*** is the process of repeating the same procedure or computation over and over again.

A **recursive formula** uses the preceding term to define the next term of the sequence. A recursive formula must state the value of the first term.

An ***explicit*** formula allows direct computation of any term for a sequence.

A sequence in which a constant is added to each term to get the next term is an ***arithmetic sequence***. The constant (d) is called the ***common difference***. To find the common difference (d), subtract any term from the term that follows it.

Arithmetic Sequence Formulas:

Recursive: $a_1 =$ _____ , $a_n = a_{n-1} + d$ *where d = the common difference*

$d = a_n - a_{n-1}$

Explicit: $a_n = a_0 + nd$ *where a_0 is the "imaginary" 0^{th} term*

-or-

$a_n = a_1 + (n-1)d$

For each of the following sequences, discover the pattern and find the next three terms of the sequence. Explain what pattern you used to find the next three terms. Write the recursive and explicit formulas for each sequence.

3, 8. 13, 18, 23, 28, ___, ___, ___

I found the next three terms by

Recursive______________________________

Explicit _______________________________

7, 3, -1, -5, ___, ___, ____

I found the next three terms by

Recursive_______________________________

Explicit _________________________________

8, 10.5, 13, 15.5 , ____, _____, ____

I found the next three terms by

Recursive_______________________________

Explicit _________________________________

-22, -12, -2, 8, ___, ___, ____

I found the next three terms by

Recursive_______________________________

Explicit ______________________________

Every week, Jane, a travel agent, gets paid \$900 (her base salary) plus an additional \$100 for each cruise she books.

(a) Complete the table below by identifying her salary based on the number of cruises she books in a week.

Cruises	Salary	Recursive Pattern
0	900	900
1	1000	900 + 100
2		1000 + 100
3		
4		

(b) What is a recursive rule for the sequence of salaries?

(c) Write an explicit rule for the sequence of salaries. Let c represent the number of cruises she books and s represent her salary.

(d) Find Jane's salary when she books 8 cruises.

Day 7: Arithmetic sequence practice

ARITHMETIC Recursive and Explicit Worksheet Name _______________

Given the following formulas, find the first 4 terms.

1. $t_1 = 0$
   $t_n = t_{n-1} + 6$

2. $t_1 = -4$
   $t_n = t_{n-1} + 2$

3. $t_1 = 8$
   $t_n = t_{n-1} - 4$

4. $t_n = 3n - 1$

5. $t_n = 4n + 3$

6. $t_n = -5n + 2$

7. Write an explicit and recursive formula for the following sequences.

a. -4, -6, -8, -10…

Explicit: _______________

Recursive: _______________

b. 84, 71, 58, 45…

Explicit: _______________

Recursive: _______________

c. 19, 13, 7, 1…

Explicit: _______________

Recursive: _______________

d. 9, 17, 25, 33…

Explicit: _______________

Recursive: _______________

e. -3, -1, 1, 3…

Explicit: _______________

Recursive: _______________

f. 110, 88, 66, 44…

Explicit: _______________

Recursive _______________

Determine if the sequence is arithmetic. If it is, find the common difference, the term named in the problem, the explicit formula, and the recursive formula.

1) $-1, -4, -16, -64, \ldots$
Find a_{39}

2) $2, 4, 12, 48, \ldots$
Find a_{22}

3) $0, -4, -8, -12, \ldots$
Find a_{38}

4) $-37, -57, -77, -97, \ldots$
Find a_{34}

Given the first term and the common difference of an arithmetic sequence find the term named in the problem, the explicit formula, and the recursive formula.

5) $a_1 = 18,\ d = -6$
Find a_{20}

6) $a_1 = 31,\ d = -200$
Find a_{20}

Given the second term and the common difference of an arithmetic sequence find the term named in the problem, the explicit formula, and the recursive formula.

7) $a_2 = -27,\ d = -9$
Find a_{40}

8) $a_2 = 206,\ d = 200$
Find a_{20}

Given the recursive formula, write the explicit formula for the sequence.

8. $t_1 = 0$
$t_n = t_{n-1} + 6$

9. $t_1 = -4$
$t_n = t_{n-1} + 2$

10. $t_1 = 8$
$t_n = t_{n-1} - 4$

Given the explicit formula, write the recursive formula for the sequence.

11. $t_n = 3n - 1$

12. $t_n = 4n + 3$

13. $t_n = -5n + 2$

Day 8: Solving Multi-Step Equations and Inequalities

Steps for Solving Equations:

1. Apply the distributive property on each side if possible.
2. Combine like terms separately on each side if possible.
3. Move the variables to the same side. Remember when moving a term to the other side, you add its opposite value.
4. Move the constants to the other side, away from the variables.
5. Solve for the variable.

Steps for Solving Inequalities:

1. Follow all the same steps for solving equations until you get to the last step.
2. REMEMBER: If you multiply or divide by a negative number to solve for the variable, you must "flip" the inequality sign!

Class Practice: solve each equation or inequality and show all work!

1. $-26 > 7 + 10n + 7$
2. $2 + 5x + 4x = 6$
3. $-298 \leq -1 + 9(2 - 7k)$
4. $-2(3 + 6y) - 9y = -132$
5. $15x + 2x = 12x - 13x - 18$
6. $2p + 3(p + 4) = 18p - p - 12$
7. Solve $D = rt + 4$ for t
8. $A = \frac{1}{2}(b_1 + b_2)h$ for h

FCC3

DAY 8 Homework NAME______________

Solve.

1. $5(x-2)-4(x+1) = 50$

2. $6x-(2x+4) = 3x$

3. $2(x+1)-(x-5) = -3(2x-5)$

4. $6(x+2)-(2x+12) = 2(4-3x)$

5. $\dfrac{x-4}{2}-\dfrac{2x-3}{4}=\dfrac{7}{4}+\dfrac{5x+1}{3}$

6. $\dfrac{1}{2}(6+4x)-\dfrac{1}{4}(8x-12)=\dfrac{1}{2}(2x-4)$

7. $5y-[7-(2y-1)] = 3(y-5)+4(y+3)$

8. $2x-5(x-2) = 1-3(x-3)$

9. $\dfrac{1}{5}(x-1)-\dfrac{1}{2}(3-2x)=\dfrac{1}{2}x+1$

10. $\dfrac{3}{4}(x-1)-\dfrac{1}{2}=2(1-3x)$

11. $5+6[5(3x+20)+2(5x+11)] = 5-2[4(x+2)+3(7x+42)]$

12. solve $P=2L+2W$ for W

13. Solve $R=5T-2Q$ for T

14. $5(2x+3) > 2(x-3)+x$

15. $4(2-x)-3(1+x) \le 5(1-x)$

16. $-\dfrac{1}{2}x-7>-5$

17. $-\dfrac{3}{5}x-1 \ge -16$

18. $3x > 6x+12$

19. $\dfrac{3x-2(x-1)}{6} \ge -1$

Answers:

1.) 64 2.) 4 3.) $\dfrac{8}{7}$ 4.) $\dfrac{4}{5}$ 5.) -2 6.) 8 7.) ϕ 8.) $\Re$ 9) $\dfrac{27}{7}$ 10.) $\dfrac{13}{27}$

11.) -5 12.) $w=\dfrac{P-2L}{2}$ 13.) $T=\dfrac{R+2Q}{5}$ 14.) $x > -3$ 15.) $x \ge 0$ 16.) $x \le -4$ 17.) $x \le 25$

18.) $x \le -4$ 19.) $x \ge -8$