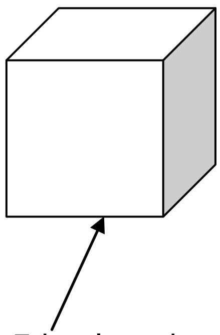
Day 8: Modeling Advanced Functions

Warm-Up:
Build or draw a set of cubes with the following edge lengths. Be sure to identify the measure used.

	Edge Length	Perimeter of one face	Surface Area of the cube	Volume of Cube
 Edge length	1 cm			
	2 cm	8 cm		
	3 cm		54 cm^2	
	4 cm			$64cm^3$
	10 cm			
	K cm			

Describe the patterns in the tables, graphs, and equations which relate edge length to perimeter, edge length to surface area and edge length to volume of a cube.

When edge length is increased by a factor of *k*, how does perimeter and volume and surface area change? Explain.

Inverse Variation

A relationship that can be written in the form $y = \frac{k}{x}$, where k is a nonzero constant and $x \neq 0$, is an <u>**inverse variation**</u>. The constant k is the constant of variation.

Multiplying both sides of $y = \frac{k}{x}$ by x gives ______________________. So, the product of x and y in an inverse variation is ___________________________.

Inverse Variations

WORDS	NUMBERS	ALGEBRA
y varies inversely as x.	$y = \frac{3}{x}$	$y = \frac{k}{x}$
y is inversely proportional to x.	$xy = 3$	$xy = k (k \neq 0)$

There are two methods to determine whether a relationship between data is an inverse variation. You can write a function rule in $y = \frac{k}{x}$ form, or you can check whether xy is a constant for each ordered pair.

Example: Tell whether the relationship is an inverse variation. Explain. If it is an inverse variation, write the equation.

1.

x	y
1	30
2	15
3	10

2.

x	y
1	5
2	10
4	20

3. $2xy = 28$

4.

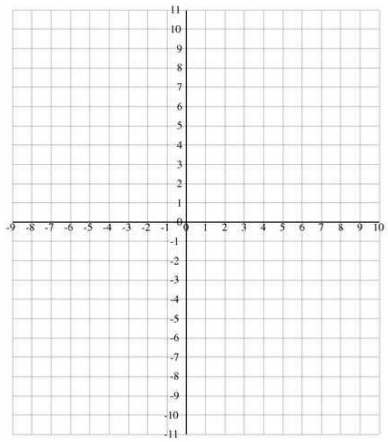
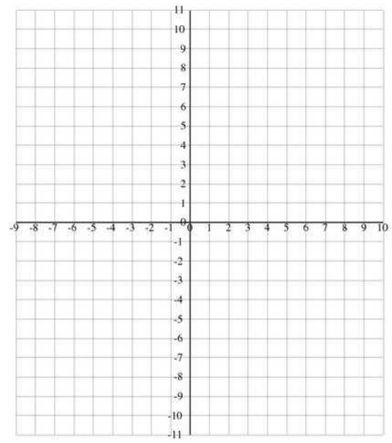
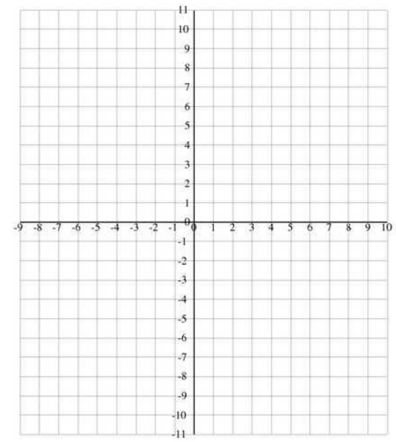
X	y
-12	24
1	-2
8	-16

5.

x	y
3	3
9	1
18	0.5

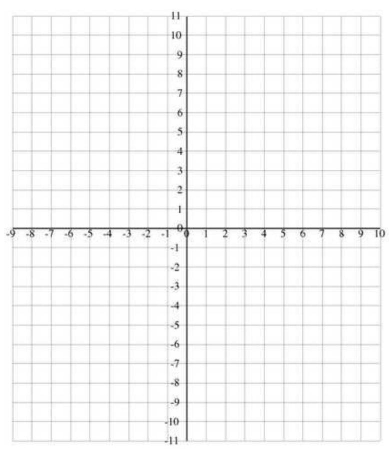
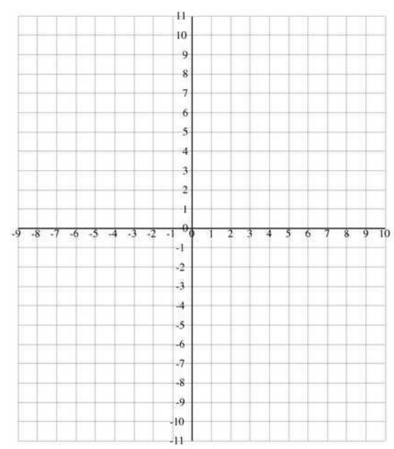
6. $2x + y = 10$

Graphing Inverse Variation…

$y = \frac{1}{x}$	$y = \frac{2}{x}$	$y = \frac{-5}{x}$
		

Things worth noting:

Examples:

1. Write and graph the inverse variation in which $y = 0.5$ when $x = -12$. 	2. Write and graph the inverse variation in which y =1/2 when $x = 10$ 

3. The inverse variation $xy = 350$ relates the constant speed x in mi/h to the time y in hours that it takes to travel 350 miles. Determine a reasonable domain and range and then graph this inverse variation.

4. The inverse variation $xy = 100$ represents the relationship between the pressure x in atmospheres (atm) and the volume y in mm^3 of a certain gas. Determine a reasonable domain and range and then graph this inverse variation.

Product Rule for Inverse Variation

If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = x_2y_2$.

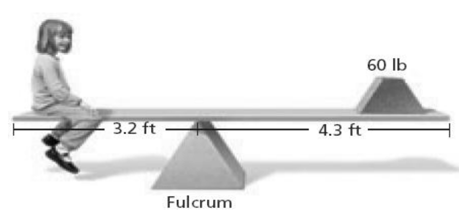
Examples

5. Let $x_1 = 5$, $x_2 = 3$, and $y_2 = 10$. Let y very inversely as x. Find y_1.

6. Let $x_1 = 2$, $y_1 = -6$, and $x_2 = -4$. Let y very inversely as x. Find y_2.

7. Boyle's law states that the pressure of a quantity of gas *x* varies inversely as the volume of the gas *y*. The volume of gas inside a container is 400 in^3 and the pressure is 25 psi. What is the pressure when the volume is compressed to 125 in^3?

8. On a balanced lever, weight varies inversely as the distance from the fulcrum to the weight. The diagram shows a balanced lever. How much does the child weigh?



Day 9: Solving Rational Equations

Warm-Up: Without a calculator!

1. Simplify: $\frac{5}{12} - \frac{1}{12} =$

3. Simplify: $\frac{4}{5} + \frac{1}{7} =$

2. Simplify: $\frac{6}{4} - \frac{3}{7} =$

4. Simplify: $\frac{2}{3} + \frac{5}{6}$

Day 9: Solving Rational Equations

A ______________________________ is an equation that contains one or more rational expressions. It can have a variable in the numerator and/or the denominator. *Our goal when solving a rational equation is to eliminate the fractions and solve the equation for the variable!*

Recall that when you graph a rational function, there is a vertical asymptote. This is an x-value that the graph *approaches* but NEVER touches. When you solve rational equations, there are some values for x that must be excluded from the domain because they will make the denominator equal to zero, and dividing by zero is undefined. Any number that causes the denominator to equal zero is called an _________________________. To find the excluded values, set the denominator equal to zero and solve for the variable; the solutions are the excluded values. When solving rational equations, if **all solutions of the rational equation are excluded values** then there is **no solution** to the rational equation*!*

To solve simple rational equations, the cross product property can be utilized to eliminate the fraction leaving a linear equation to solve. **REMEMBER:** Check your final answers to make sure they are not an excluded value*!*

Examples: Using the cross product property, solve the following equations. Do not forget to determine the excluded values.

1. $\frac{6}{x} = \frac{3}{7}$ EV: __________

2. $\frac{4}{x-7} = \frac{6}{x}$ EV: __________

3. $\frac{-5}{x+4} = \frac{1}{x+4}$ EV: __________

4. $\frac{6}{x+5} = \frac{x}{6}$ EV: __________

Examples: Multiply through by the LCD to solve the following equations. Do not forget to determine the excluded values.

5. $\frac{2}{x} - 3 = \frac{8}{x}$ EV: __________

6. $\frac{7x}{x-3} + 4 = \frac{x+1}{x-3}$ EV: __________

You Try!

Examples: Solve the rational equation. Do not forget to determine the excluded values.

7. $\frac{8}{x+8} = \frac{x}{x+2}$ EV: __________

8. $\frac{4}{x+2} + 3 = \frac{9}{x+2}$ EV: __________

9. $\frac{3x}{x-1} - 2 = \frac{10}{x-1}$ EV: __________

10. $\frac{12}{x+2} = \frac{7}{x-3}$ EV: __________

Solving Simple Rational Equations Practice

Solve the rational equation. Do not forget to determine the excluded values.

1. $\frac{3}{x} = \frac{2}{x+4}$ EV: __________

2. $\frac{x+1}{2x+5} = \frac{2}{x}$ EV: __________

3. $\frac{3}{x+2} + 5 = \frac{4}{x+2}$ EV: __________

4. $\frac{6}{x-3} = \frac{x}{18}$ EV: __________

5. $\frac{2x}{x+4} - 3 = \frac{-12}{x+4}$ EV: __________

6. $\frac{14}{2-x} = \frac{2}{x}$ EV: __________

Day 10: Solving Harder Rational Equations

Warm-up:

1. $\frac{x+2}{x+1} - x = \frac{-6}{x+1}$ EV: __________

2. $\frac{4}{x-5} = \frac{2}{x+8}$

3. $\frac{2}{x-4} + 2 = \frac{6}{x-4}$ EV: __________

4. $\frac{x}{x+24} = \frac{2}{x}$

Day 10: Solving Harder Rational Equations

Example 1: $\frac{x-4}{4} + \frac{x}{3} = 6.$

Steps:
1. Find the LCD.
2. Multiply each side by the LCD.
3. Simplify.
4. Solve for x!

$$\frac{x-4}{4}+\frac{x}{3}=6.$$

$$12\left(\frac{x-4}{4}+\frac{x}{3}\right)=12(6)$$

$$3(x-4)+4(x)=72$$

$$3x-12+4x=72$$

$$7x=84$$

$$x=12$$

The LCD of the fraction is 12.

Multiply each side of the equation by 12. The fractions are eliminated.

Emphasize that each term must be multiplied by the LCD in order to have a balanced equation. A common mistake is to multiply only those terms that are expressed in fractions.

Check $\rightarrow \frac{x-4}{4}+\frac{x}{3}=6 \rightarrow \frac{12-4}{4}+\frac{12}{3}=6 \rightarrow 2+4=6 \rightarrow 6=6$

Example 2: $$\frac{3}{2x}-\frac{2x}{x+1}=-2$$

Note that $x \neq -1$ and $x \neq 0$. The LCD of the fractions is $2x(x+1)$

Multiply each side of the equation by $2x(x+1)$.

Example 3: $$\frac{k+1}{3}-\frac{k}{5}=3$$

Example 4: $$\frac{6}{x}-\frac{9}{x-1}=\frac{1}{4}$$

Example 5: $$\frac{2m}{m-1}+\frac{m-5}{m^2-1}=1$$

Solving Rational Equations Practice

1. $\frac{2a-3}{6}=\frac{2a}{3}+\frac{1}{2}$

2. $\frac{2b-3}{7}-\frac{b}{2}=\frac{b+3}{14}$

3. $\frac{3}{5x}+\frac{7}{2x}=1$

4. $\frac{5k}{k+2}+\frac{2}{k}=5$

5. $\frac{m}{m+1}+\frac{5}{m-1}=1$

6. $\frac{4x}{3x-2}+\frac{2x}{3x+2}=2$

7. $\frac{5}{5-p}-\frac{p^2}{5-p}=-2$

8. $\frac{2a-3}{a-3}-2=\frac{12}{a+3}$

9. $\frac{2b-5}{b-2}-2=\frac{3}{b+2}$

10. $\frac{4}{k^2-8k+12}=\frac{k}{k-2}+\frac{1}{k-6}$

Day 11: Advanced Functions Review

Warm-up:

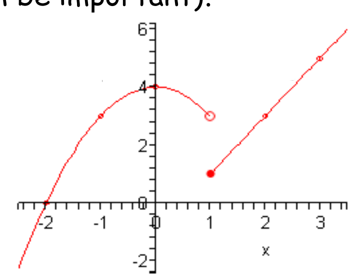
Find the domain and range of the following functions. Then, tell how they are changed from their parent graph. (Hint: Remember that the order of transformations can be important).

1) $f(x)=2[x+3]-4$

2) $f(x)=\sqrt[3]{8x-16}-5$

3) $f(x)=-\sqrt{9x+54}+2$

4) $f(x)=-3|x-7|+1$



5) Write a Piecewise Function for the graph shown. Then tell its domain and range. (Hint: use graph paper!)