# Foundations of Common Core Math 3

## Unit 4 – Polynomials


**"It's important to learn math because someday you might accidentally buy a phone without a calculator."**

**Name:** _____

WAKE COUNTY
PUBLIC SCHOOL SYSTEM

APEX HIGH SCHOOL
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FCC3 -- Unit 4: Polynomials

| Day | Classwork | Homework |
|---|---|---|
| Day 1: Thurs 10/23 | Monomial and Exponent review: notes, wksht #1 | Worksheet #2 |
| Day 2: Fri 10/24 | More monomial practice Wksht #3 then QUIZ 1 | |
| Day 3: Mon 10/27 | Factoring GCF, Diff of Squares<br>Factoring practice #4 evens | Practice #4odds |
| Day 4: Tues 10/28 | Factoring Trinomials and grouping Practice#5 and #6 evens | Practice #5 and #6 odds |
| Wed 10/29 | TEACHER WORKDAY | END OF QUARTER 1 |
| Day 5: Thurs 10/30 | Factoring cubes, combining methods<br>Practice #7 evens | Practice #7 odds |
| Day 6: Fri 10/31 | Group practice #8<br>QUIZ #2: FACTORING | |
| Day 7: Mon 11/3 | Polynomial division—synthetic and long<br>Both wkshts #9 and #10 evens | Wkshts 9 and #10 odds |
| Day 8: Tues 11/4 | Polynomials, end behavior, forms, zeros<br>Practice #11 evens | Practice page #11 odds |
| Day 9: Wed 11/5 | More polynomial zeros, roots, factors sketches<br>Practice #12 evens | Practice #12 odds |
| Day 10: Thurs 11/6 | Sketch graph from roots, write eq. from roots<br>Wksht #13 evens | wksht #13odds |
| Day 11: Fri 11/7<br>Early release | Quiz #3 | Review #1 |
| Day 12: Mon 11/10 | Review #2 | NO SCHOOL Tuesday 11/11<br>STUDY FOR TSET |
| Day 13: Wed 11/12 | Unit 4 TEST | |

Unit 4 Notes FCC3

I. Monomial – is an expression that is a number, a variable, or the product of a number and one or more variables.

Ex: 3a, -5, $t^4$, $\frac{2}{3}ab^2$

Constants- monomials that contain no variables. Ex: 4 in $x^2 + x + 4$

Coefficient- the numerical factor of a monomial. Ex: 3 in $3x^2$

Degree of a monomial- the sum of the exponents.

Ex: $5x^3y^2z^4$ the degree $= 3 + 2 + 4 = 9$

Degree of a polynomial- the highest total of a monomials degree.

Ex: $5x^3y^2 + 3x^3y^4 - 75x^2y$ the degree = 7

II. For any real numbers a,b and positive integers m,n

1) $a^m \cdot a^n = a^{m+n}$ Ex: $3^2 \cdot 3^3 = 3^5$
2) $(a^m)^n = a^{mn}$ Ex: $(4^2)^3 = 4^6$
3) $(ab)^m = a^m b^m$ Ex: $(3 \cdot 4)^2 = 3^2 4^2$

III. RULES: For any real number a, except a = 0 and integers m,n

1) $\frac{a^m}{a^n} = a^{m-n}$ Ex: $\frac{4^5}{4^2} = 4^{5-2} = 64$

2. $a^{-n} = \frac{1}{a^n}$ Ex: $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

3. $\frac{1}{a^{-n}} = a^n$ Ex: $\frac{1}{6^{-2}} = 6^2 = 36$

4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ or $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

IV. Scientific Notation

$.00008 = 8 * 10^{-5}$ move right
$2{,}340{,}000 = 2.34 * 10^6$ move left

***Worksheet day 1*** Name_____

Simplify.

1. $3n^2v^3 - n^2v^3 + 8v^3n^2$

2. $4r^6w^2 + 9r^2w^6 - r^6w^2$

3. $y^7 \bullet y^3 \bullet y^2$

4. $(n^6)^3$

5. $(2n)^4 + 2n^4$

6. $(3r^7t^2)(-5rt^9)$

7. $(4a^3c^2)^3(-3ac^4)^2$

8. $\left(\frac{3}{2}e^2f^4\right)^4\left(-\frac{4}{3}e^5f\right)^3\left(-\frac{1}{6}ef^5\right)$

9. $-5v^2(2r^3v^2)(rv^3) - (-r^2)(16r^2v^7)$

10. $(-n)^4(2xy^2n)^3 + (4xy^3n^2)^2(-3xn^3)$

11. $(3b^2)^4(-2b^3)^8$

12. $(m^4n^6)^4(m^3n^2p^5)^6$

13. $(3x^2y)(2xy^4) + (4xy^4)(3x^2y^3)$

14. $(7v^3w^4)(2v^2w^6) + (3vw^5)(2v^4w^5)$

Evaluate. Express each answer in both scientific and decimal notation.

15. $(2.3\times10^4)^2$

16. $(8.7\times10^3)^2$

17. $(4.8\times10^2)\ (6.9\times10^4)$

18. $(3.7\times10^9)\ (8.7\times10^2)$

19. (46,000)(0.025)

20. (54,000)(0.00073)

***day 1 worksheet 2*** Name_____

Simplify. Assume no variable equals zero.

1. $p^{-7}p^{3}$

2. $\frac{21y^{9}}{35y^{3}}$

3. $\frac{12m^{8}}{-9my^{4}}$

4. $\frac{14(x^{-3})^{4}}{(2x^{2})^{3}}$

5. $\frac{5x^{4}}{(6x^{7})^{0}}$

6. $\frac{-6n^{9}t^{3}}{-18n^{9}t^{5}}$

7. $\frac{-20(m^{2}v)(-v)^{3}}{5(-v)^{2}(-m^{4})}$

8. $\frac{x^{7y+1}}{x^{7y-5}}$

9. $\frac{(3m)^{-4}}{2m^{-4}}$

10. $(x^{7}y^{3})^{-3}$

11. $\left(\frac{3}{8}\right)^{-2}x^{8}y^{-3}z$

12. $\frac{5m^{3}}{t^{-6}}$

13. $\frac{6^{-2}x^{4}}{2^{-3}x^{-3}}$

14. $\frac{(3x^{-2}y^{3})(5xy^{-8})}{(x^{-3})^{4}y^{-2}}$

15. $t^{-5}(t^{2}-t^{4}+5t)$

16. $\frac{(8c^{2})^{-2}}{c^{4}}$

17. $\left(\frac{5}{n}\right)^{-4}$

18. $\left(\frac{x}{r^{-2}}\right)^{-3}$

19. $\left(\frac{12x^{-2}}{4x^{6}}\right)^{-1}$

20. $\left(\frac{4}{9}\right)^{-3}\left(\frac{4}{9}\right)^{5}$

21. $\left(\frac{1}{2ay^{-3}}\right)^{-2}$

22. $\left(\frac{6}{2ay^{2}}\right)^{3}$

23. $\left(\frac{1}{8}\right)^{-2}-\left(\frac{1}{2}\right)^{-4}$

24. $\left(\frac{n^{3}}{(n^{2})^{-2}}\right)^{-1}$

Evaluate. Express each answer in both scientific and decimal notation.

25. $\frac{4\times10^{8}}{1.6\times10^{4}}$

26. $\frac{2.7\times10^{6}}{9\times10^{10}}$

***day 1 wksht 2*** Name_____

Find each product.

1. $(x + 3)(2x – 5)$

2. $(x^2 + x – 1)(x + 1)$

3. $(3w + 4)(2w – 1)$

4. $(x + 5)(x + 4)$

5. $(2b – 1)(b^2 – 3b + 4)$

6. $(a – 11)(a + 5)$

7) $(2g – 3)(2g^2 + g – 4)$

8) $(3s – 4)(s – 5)$

9) $(4x + 3)(x – 7)$

10) $(x + 6)(x^2 – 4x + 3)$

11) $(5x – 3)(4x + 2)$

12) $(3y + 7)(4y + 5)$

13) $(3x + 7)(x + 5)$

14) $(5x – 2)(x + 3)$

15) $(3m^2 – 7m + 8)(m – 2)$

16) $(a – 6)(a + 8)$

17) $(x + 2)(2x^2 – 3x + 2)$

18) $(a^2 + a + 1)(a – 1)$

19) $(x – 2)(x^2 + 4x + 4)$

20) $(2r + 1)(3r – 1)$

21) $(k + 4)(3k – 4)$

22) $(2n – 3)(n^2 – 2n + 5$

23) $(p – 4)((2p + 3)$

24) $(3x + 1)(4x^2 – 2x + 1)$

25) $(2x^2 – 5x + 2)(4x – 3)$

26) $(x + 7)(x + 5)$

27) $(6x – 7)(x - 2)$

28) $(2x + 1)(4x + 3)$

29) $(3x + 4)(3x – 4)$

30) $(6x – 11)(x + 2)$

FCC3 2 Day 2 wksht 3 Monomials

Simplify.

1. $3x + 2x + (-4x)$

2. $4d^3 - d^3 + 2d^3$

3. $4ab^2 - 3ab^2$

4. $3x^2 + 4 - 3x^2$

5. $y^5 \cdot y^7$

6. $b^4 \cdot b^3 \cdot b^2$

7. $8^6 \cdot 8^4 \cdot (8^2)^2$

8. $(y^5)^2$

9. $(3a)^4$

10. $(x^2y^2)^2(x^3y^3)$

11. $\left(-\frac{3}{4}x^2y^3\right)^2\left(\frac{8}{9}xy^4\right)$

12. $\left(\frac{3}{5}c^2f\right)\left(\frac{4}{3}cd\right)^2$

13. $(-4a)(a^2)(-a^3) + 3a^2(a^4)$

14. $2(rk)^2(5rt^2) - k(2rk)(2rt)^2$

15. $(5a)(6a^2b)(3ab^3) + (4a^2)(3b^3)(2a^2b)$

16. $(5mn^2)(m^3n)(-3p^2) + (8np)(3mp)(m^3n^2)$

17. $t^{-2}t^4$

18. $m^{-8}m^3$

19. $\frac{12x^8}{4x^3}$

20. $\frac{an^6}{n^5}$

21. $\frac{-24s^8}{2s^5}$

22. $\frac{6mn^2}{3m}$

23. $\frac{xy^7}{x^4}$

24. $\frac{48a^8}{12a^{11}}$

25. $\frac{4z^3}{28z^5}$

26. $\frac{-15r^4}{30r^3}$

27. $\frac{12b^4}{60b^6}$

28. $\frac{2x^{-3}}{6(x^2)^2}$

29. $\frac{16b^6c^5}{4b^2c^2}$

30. $\frac{8(k^{-2})^2}{4k^{-2}}$

31. $\frac{1}{x^0+y^0}$

32. $\frac{-27w^3t^7}{-3w^3t^{12}}$

33. $\frac{-15r^5s^2}{5r^5s^{-4}}$

34. $\frac{8}{m^0+n^0}$

35. $\frac{-2c^3d^6}{24c^2d^2}$

36. $\frac{(3c^2)^2(-d^5)}{-45c^7d^3}$

37. $\frac{20a^5b^9}{20ab^7}$

38. $\frac{16s^6t^5}{(2s^2t)^2}$

39. $\frac{3^{xy+5}}{3^{xy}}$

40. $\frac{s^{3x}}{s^{3x-2}}$

41. $(m^4n^5)^{-2}$

42. $36a^3b^5(12a^2b^2)^{-1}$

43. $\left(\frac{a}{b^{-1}}\right)^{-1}$

44. $\left(\frac{x}{y^{-1}z^2}\right)^{-1}$

45. $\left(\frac{-3y^4}{2y^2}\right)^{-2}$

46. $\left(\frac{5y^{-3}}{4y^{-6}}\right)^{-2}$

Day 3 Practice #4 Factoring GCF and Difference of Squares

Complete.

1. $8m - 6 = 2(4m - \_\_\_\_\_)$

2. $36a^2 + 24b^2 = 12(\_\_\_\_\_ + 2b^2)$

3. $12x^3y - 15xy = \_\_\_\_\_ (4x^2 - 5)$

4. $5a^2b - 10a^2b^2 = \_\_\_\_\_ (1 - 2b)$

Factor each polynomial.

1) $5a^2 - 15$

2) $7x + 49$

3) $2y + 6xy$

4) $8ax - 56a$

5) $36xy^2 - 48x^2y$

6) $75b^2c^3 + 60bc^6$

7) $64 - 40ab$

8) $81 - 36xy$

9) $t^2h + 3t$

10) $6p - 72$

11) $81r + 48rs$

12) $5c^3 - 2c^2$

13) $82e^2 - 122ef$

14) $10q - 25q^2$

15) $xy^2 + xy$

16) $15cd + 30c^2d^2$

17) $a^2b^2 + a$

18) $6r^2s - 3rs^2$

19) $l^2 - 9l$

20) $4d^2 + 16$

21) $6z^4 - 18z^3$

22) $20p^2 - 16p^2q^2$

23) $6m^4 - 60$

24) $7a^3 + 14a^2$

25) $16wv^4 + 12w^3v^2$

26) $9c^4d^3 - 6c^2d^4$

27) $6y + 15y^2$

28) $30x^3y + 35x^2y^2$

29) $6e^2f - 11ef$

30) $20r^3s^2 + 25rs^3$

31) $34x^4y^3 - 17x^2y^5$

32) $35m^3n + 105m^2n^3$

33) $2d^2e^2 - 8d^6e^6$

34) $2c^2 - 24c + 54$

35) $162 - 45x + 3x^2$

36) $32a + 16a^2 + 2a^3$

Day 4: Factoring Practice #5 : FACTORING BY GROUPING

1.) $ax + ay + bx + by$

2.) $8x^2 + 2xy + 12x + 3y$

3.) $6mn - 9m - 4n + 6$

4.) $2x^2y + 6xy - x - 3$

5.) $4k + 12 + k^2 + 3k$

6.) $p^2q + pq - 1 - p$

7.) $2ac + ad + 6bc + 3bd$

8.) $4r^2s - 8rs - 3r + 6$

9.) $z^3 - 6 + 2z - 3z^2$

10.) $3a - 5a^2 - 6b + 10ab$

*Day 4: Practice* #6 – FACTORING BY NUMBER GAME AND "TRIAL AND ERROR"

1) $a^2 + 3a + 2$

2) $a^2 + 4a + 3$

3) $a^2 + 5a + 6$

4) $a^2 + 10a + 21$

5) $x^2 + 12x + 27$

6) $x^2 - 11x + 24$

7) $x^2 - 9x + 8$

8) $x^2 + 9x - 10$

9) $x^2 - 19x - 20$

10) $x^2 + x - 12$

11) $a^2 - a - 56$

12) $y^2 - y - 2$

13) $x^2 - x - 20$

14) $a^2 + 5a - 6$

15) $15 + 16a + a^2$

16) $x^2 - 5x - 24$

17) $56 - 30a + a^2$

18) $5a^2 - 7a + 2$

19) $4x^2 - 11x + 6$

20) $2y^2 - 5y + 2$

21) $5a^2 + 4a - 1$

22) $7x^2 + 19x - 6$

23) $2x^2 + x - 15$

24) $10a^2 + 3a - 4$

25) $25c^2 + 10cd - 8d^2$

26) $5a^2 + 33a - 56$

27) $12x^2 + 19xy - 10y^2$

28) $10a^2 + a - 2$

29) $8x^2 + 10x - 3$

30) $12x^2 + 79x - 35$

31) $2a^2 - 3a - 14$

32) $16x^2 - 6x - 27$

33) $12x^2 - 5xy - 2y^2$

34) $6a^2 - a - 40$

35) $9x^2 - 3x - 2$

36) $8y^2 - 18y - 5$

37) $12x^2 - 53x - 35$

38) $12x^2 - 4xy - 21y^2$

39) $4a^2 - 4a - 15$

40) $10 - 3x - 27x^2$

***Day 5: Practice #7***

Sum and difference of Cubes: You must memorize the patterns for this factoring method.

Ex 1.) $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$ Ex 2.) $y^6 + 8 = (y^2 + 2)(y^4 - 2y^2 + 4)$

Short factor: cube root, keep the sign, cube root

Long factor: square the first term, change the sign, multiply the 2 terms together, square the second term

Practice all the methods:

1) $x^2 - 9$

2) $4m^2 - 1$

3) $4x^2 + 12x + 9$

4) $n^2 - 4$

5) $16c^2 - 49$

6) $4x^2 - 20x + 25$

7) $x^2 - 24x + 144$

8) $9a^2 + 60a + 100$

9) $81x^2 - 400$

10) $3a^2 - 48$

11) $10x^3 - 80$

12) $12w^2 - 27$

13) $x^2 + 6x + 9$

14) $36s^2 - 225$

15) $x^2 - 16x + 64$

16) $d^2 - 49$

17) $9y^2 - 289$

18) $100a^2 - 9$

19) $5n^2 - 20n$

20) $d^2 - 169$

21) $x^2 - 121$

22) $16n^2 + 56n + 49$

23) $a^2 + 26a + 169$

24) $9d^2 - 64$

25) $49a^2 - 14a + 1$

26) $8x^6 - 1$

27) $4x^2 - 60x + 225$

28) $y^2 - 81$

29) $256a^2 - 1$

30) $2d^3 - 50d$

31) $64y^3 + 27$

32) $x^2 - 12x + 36$

33) $50m^3 - 32m$

34) $x^2 - 18x + 81$

35) $16x^2 - 72x + 81$

36) $x^2 + 24x + 144$

37) $2x^3 + 40x^2 + 200x$

38) $16x^2 + 8x + 1$

39) $x^2 + 30x + 225$

40) $9r^2 - 256$

41) $m^3 + 125$

42) $x^3 - 27$

43) $25x^2 - 30x + 9$

44) $9x^2 - 400$

45) $64x^2 + 80x + 25$

**Group member names:**_____

## Day 6: GROUP Factoring Practice #8

Make sure to factor completely!!

1.) $4w^2 + 5w - 21$

2.) $20b^4 - 45c^2$

3.) $15x^4 + 3x^2 - 18$

4.) $8n^3 - 27y^9$

5.) $5b^4 - 15b^3 + 10b^2$

6.) $48 - 2k - k^2$

7.) $x^4 - y^4$

8.) $3x^3 - 27x + 2x^2y - 18y$

9.) $25m^4 - 100\ m^2$

10.) $y^6 - x^6$

11.) $x^{2n} + 3x^n - 10$

12.) $2x^{10} - 2x^5 - 60$

FCC3 day 7 Name_____
Division Worksheet #9 Long Division with Polynomials

Simplify.

1. $(-30x^3y + 12x^2y^2 - 18x^2y) \div (-6x^2y)$

2. $(2x^2 + 3x - 4) \div (x - 2)$

3. $(4x^2 - 2x + 6)(2x - 3)^{-1}$

4. $(x^4 - 3x^3 + 5x - 6) \div (x + 2)$

5. $(6x^2 - x - 7) \div (3x + 1)$

6. $(2x^3 + 4x - 6) \div (x + 3)$

7. $(4x^3 - 8x^2 + 3x - 8) \div (2x - 1)$

8. $(x^4 - 2x^3 + 6x^2 - 8x + 10) \div (x + 2)$

Day 7: Practice #10 Division using synthetic division.

1. $(3y^3 + 2y^2 – 32y + 2) / (y – 3)$

2. $(2b^3 + b^2 – 2b + 3) / (b + 1)$

3. $(2c^3 – 3c^2 + 3c – 4) / (c – 2)$

4. $(3x^3 – 2x^2 + 2x – 1) / (x – 1)$

5. $(t^4 – 2t^3 + t^2 – 3t + 2) / (t – 2)$

6. $(3r^4 – 6r^3 – 2r^2 + r – 6) / (r + 1)$

7. $(z^4 – 3z^3 – z^2 – 11z – 4) / (z – 4)$

8. $(2b^3 – 11b^2 + 12b + 9) / (b – 3)$

9. $(6s^3 – 19s^2 + s + 6) / (s – 3)$

10. $(x^3 + 2x^2 – 5x – 6) / (x – 2)$

11. $(x^3 + 3x^2 – 7x + 1) / (x – 1)$

12. $(n^4 – 8n^3 + 54n + 105) / (n – 5)$

13. $(2x^4 – 5x^3 + 2x – 3) / (x – 1)$

14. $(z^5 – 6z^3 + 4x^2 – 3) / (z – 2)$

15. $(y^4 + 3y^3 + y – 1) / (y + 3)$

16. $(4s^4 – 5s^2 + 2s + 3) / (2s – 1)$

17. $(2x^3 – 3x^2 – 8x + 4) / (2x + 1)$

18. $(4x^4 – 5x^2 – 8x – 10) / (2x – 3)$

19. $(6j^3 – 28j^2 + 19j + 3) / (3j – 2)$

20. $(y^5 – 3y^2 – 20) / (y – 2)$

Unit 4 – Day 8
POLYNOMIAL FUNCTIONS

Polynomial Function: in the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where $a_n \neq 0$,
Coefficients ($a_n$, $a_{n-1}$,… $a_1$, $a_0$) represent real numbers, and exponents are all whole numbers.

These are polynomials in one variable

1. $6x^4 + 3x^2 + 4x - 8$ (degree 4)
2. $x + 8$ (degree 1)
3. $-5$ (degree 0)
4. $x^5 - 3x^3 - 7$ (degree 5)

**Not** a polynomial in one variable

1. $9x^3y^5 + 2x^2y^6 - 4$ (2 variables)
2. $x^{-3} + 4x^2 - 1$ (negative exp)
3. $5x^7 + 3x^2 + \frac{2}{x}$ (variable in denominator – really a negative exponent)

FINDING ZEROS

Remember that the solutions are called "zeros" and "roots"and are also the x – intercepts of the graph.

If the polynomial is written in factored form, you set each factor equal to zero and solve for x.

EXAMPLE: $y = 2x(x+3)(x-4)$ has zeros at $x = 0, -3$ or $4$

Polynomials can also be written in standard form like this.

EXAMPLE: $y = 2x^3 - 2x^2 - 24x$

In order to find the zeros, you need to factor $y = 2x(x^2 - x - 12)$

$Y = 2x(x - 4)(x + 3)$

So the zeros are $x = 0, 4$ or $-3$

END BEHAVIOR: if the degree is EVEN, both ends have SAME behavior

If "a" is positive, both ends are up

If "a" is negative, both ends are down

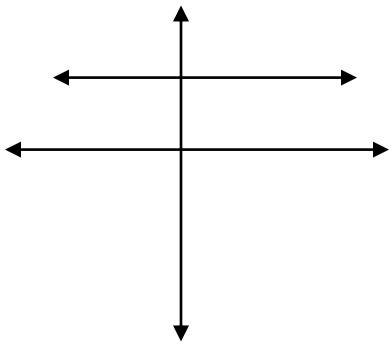
If the degree is odd, the ends have OPPOSITE behavior

If "a" is positive, the right end is up, left down

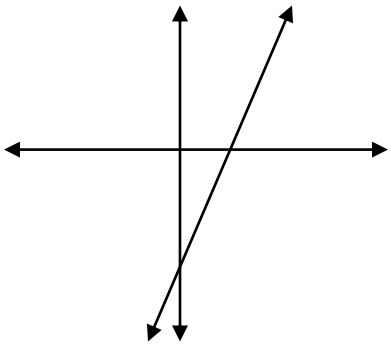
If "a" is negative, the right end is down, left up

Looking at the graphs of polynomial functions:

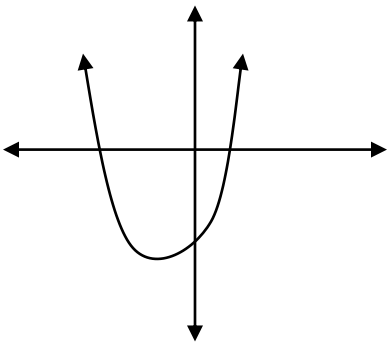
1.)Constant function
$y = 2$
Degree 0



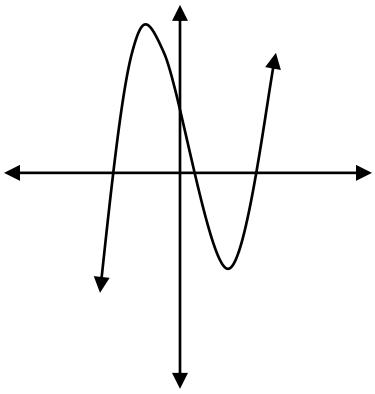
2.)Linear function
$y = 3x - 5$
Degree 1



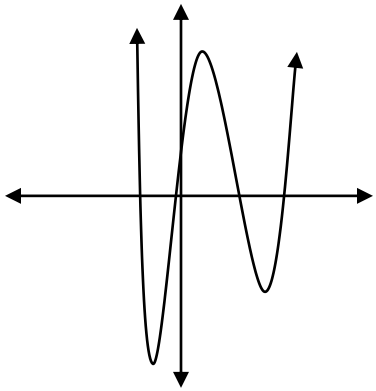
3.)Quadratic function
$y = x^2 + 2x - 3$
Degree 2



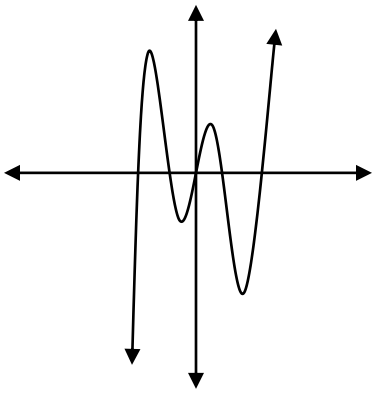
4.)Cubic function
$Y = x^3 - 5x + 2$
Degree 3



5.)Quartic function
$y = x^4 - 3x^3 - 2x^2 + 7x + 1$
Degree 4



6.)Quintic function
$y = x^5 - 5x^3 + 4x$
Degree 5



Lets look at each graph and make a prediction concerning the # of roots and the end behavior for each, then confirm our predictions by looking at the graph.

1.) constant…..degree 0 … means NO real zeros and cannot discuss end behavior because it is simply a horizontal line.
2.) linear….degree 1…..means 1 real root and since "a" is 3, it is positive, so the right end is up and the left end is down.
3.) quadratic….degree is 2….means at most 2 real roots and since "a' is 1, both ends are up.
4.) cubic….degree 3……means at most 3 real roots and since "a" is 1, the right end is up, left down.
5.) quartic…degree 4…means at most 4 real roots and since "a" is 1, both ends are up.
6.) quintic……degree 5……means at most 5 real roots and since "a" is 1, the right end is up and left down.

Unit 4 Day 8 Homework Practice #11

Determine whether the function is a polynomial. If it is, write it in standard form, then classify it based on the number of terms and its degree.

1. $12 - 5x^2$

2. $3x^3 - 2x^4 + x^5$

3. $\frac{2}{x} + x - 3$

4. $-x^3 + 2x^2 - 6x - 5$

Fill in the chart describing the end behavior.

| function | As $x \to -\infty$ | As $x \to \infty$ |
|---|---|---|
| 5. $f(x) = -5x^3 + 2x - 1$ | | |
| 6. $f(x) = 2x^4 - 3x^2 - 6$ | | |
| 7. $f(x) = -3x^2 - x + 3$ | | |
| 8. $f(x) = 2x^3 + 4$ | | |
| 9. $f(x) = -x^2 + 4x - 9$ | | |
| 10. $f(x) = -3x^4 + 2x^3 - x^2$ | | |
| 11. $f(x) = 4 - x^5$ | | |
| 12. $f(x) = 2 + 3x - x^2$ | | |
| 13. $f(x) = -2x + 4$ | | |

Name the real zeros for each function. Include multiplicity where appropriate.

14. $f(x) = x^3 - 2x^2 - 9x + 18$

15. $f(x) = x^4 + 2x^2 - 24$

16. $f(x) = x^3 + 7x^2 - 4x - 28$

17. $f(x) = x^3 - 3x^2 + 4x - 12$

18. $f(x) = -3(x+2)^2 (x-1)$

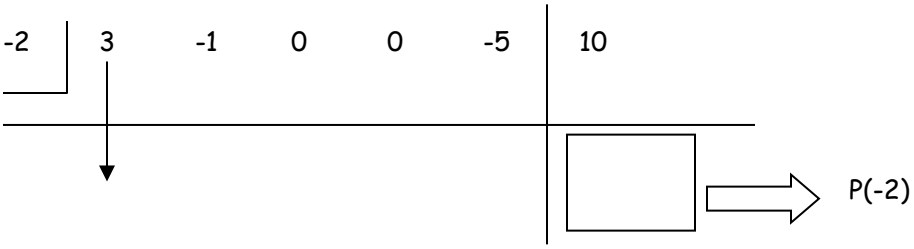
19. $f(x) = -5x^2 (x+4) (x+1)$

Use graph paper and sketch each of the functions 14 through 19.

DAY 9: More Polynomials: Roots, Factors, Zeros

Remainder Theorem: If a polynomial P(x) is divided by x – a, then the remainder = P(a).

1) Example: If $P(x) = 3x^5 - x^4 - 5x + 10$ is divided by (x+2), then the remainder is P9-2). Find P(-2) using synthetic division.

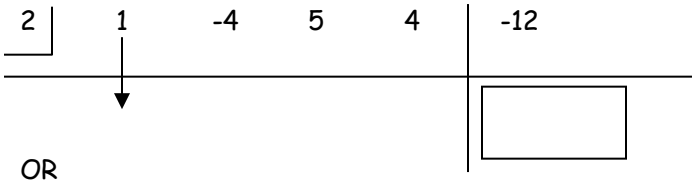



Substituting –2 for x will give the same result!

2) If $f(x) = 2x^4 - 8x^2 + 5x - 7$, find f(3) using division and substitution.

Factor Theorem: A polynomial P(x) has a factor x – a if and only if P(a) = 0.

3) Is x – 2 a factor of $f(x) = x^4 - 4x^3 + 5x^2 + 4x - 12$? YES or NO?




OR

Find f(2), if it equals zero, then x – 2 is a factor!

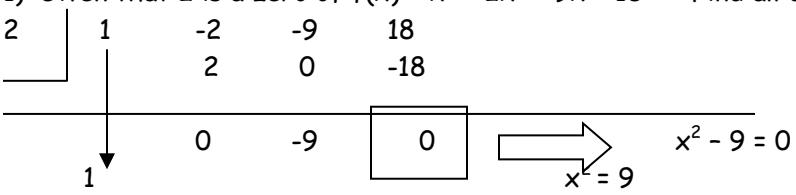
4) Find all the factors of $f(x) = 2x^3 + 11x^2 + 18x + 9$ given that f(-3) = 0. (**means that x + 3 is a factor)

5) Given that x – 2 is a factor, finish factoring $f(x) = 3x^3 + 14x^2 - 28x - 24$ completely.

## Finding Zeros

Zeros are solutions to the equation, also known as roots. When the function is graphed, the zeros of the function will be the x-intercepts of the graph.

1) Given that 2 is a zero of $f(x) = x^3 - 2x^2 - 9x + 18$ Find all other zeros.




$x^2 - 9 = 0$

$x = 3, -3$

2) Solve given that $-\frac{1}{2}$ is a root of $f(m) = 2m^3 - 5m^2 - 13m - 5$.

3) Solve $p(x) = x^3 + x + 10$, if -2 is a root.

Fundamental Theorem of Algebra: Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers (includes real numbers).

Therefore, a polynomial equation of the form $P(x) = 0$ of degree n, has exactly n roots.
# of solutions = degree of the polynomial

Every polynomial of ODD degree (with real coefficients) has at least one REAL root

Ex. $x^3 + 4x^2 + 4x = 0$ find 3 solutions Ex. $x^4 - 10x^2 + 9 = 0$ find 4 solutions

*imaginary roots occur in CONJUGATE PAIRS (if polynomial coefficients are real numbers)
Ex. If $2 + 3i$ is a root, then so is $2 - 3i$

**Multiplicity: how many times a solution repeats.
Ex. $x^2(x - 4)(x - 4)(x - 4) = 0$ Solutions: {0(multiplicity of 2), 4(multiplicity of 3)}

WRITING AN EQUATION GIVEN THE ZEROS: Use real # solutions to write factors, then use the imaginary solution to write another factor using the sum/product rule, the multiply factors.

1.) Find a cubic equation with integral coefficients that has 2 and $3 - i$ as roots.

2.) Given {4, -1, 3i} write an equation with integral coefficients.

Day 9 – Homework Practice #12

For each polynomial, show how to find f(-3) using a.) synthetic substitution and b.) synthetic division:

1.) $f(x) = 3x^3 + 4x^2 - 5x + 1$ a.) b.)

2.) $f(x) = -4x^4 - 3x^2 - 5$ a.) b.)

3.) Is (x+2) a factor of $x^4 - 4x^3 + 5x^2 + 4x - 12$? Show work to justify your answer.

4.) Is (x - 3) a factor of $2x^3 + 11x^2 + 18x + 9$? Show work to justify your answer.

5.) Factor $f(x) = 9x^3 + 6x^2 - 3x$ if you know (x+1) is a factor.

6.) Factor $f(x) = x^3 - 2x^2 - 9x + 18$ if you know (x+3) is a factor.

7.) Factor $y = x^3 - 4x^2 - 3x + 18$ if you know that (x+2) is a factor.

8.) SOLVE $x^3 + 4x^2 - 5x = 0$ completely.

9.) SOLVE $x^4 + 7x^2 - 18 = 0$ completely.

10.) List all zeros and multiplicities for $y = 2x(x+3)^2(4x - 1)$

11.) Write a cubic equation having zeros 2, $\frac{3}{4}$ and -1.

12.) Write the quartic equation having zeros $2i$ and $3-i$.

13.) Write the cubic equation having zeros $\frac{-2}{3}$ and $2 + 3i$

14.) SOLVE $x^3 - 4x^2 + 6x - 4 = 0$ given that $1 + i$ is a root.

15.) SOLVE $x^4 + x^3 + 6x^2 - 14x - 20 = 0$ knowing $-1 - 3i$ is a root.

**Answers**

1. f(-3) = -29 2. f(-3) = -356 3. No, rem ≠ 0 4. No, rem ≠ 0 5. 3(x+1)(3x-1)

6. (x+3)(x-2)(x-3) 7. $(x-3)^2(x+2)$ 8. x = 0, -5, 1 9. $x = \pm\sqrt{2}, \pm 3i$ 10. 0, ¼, -3 with mult. of 2

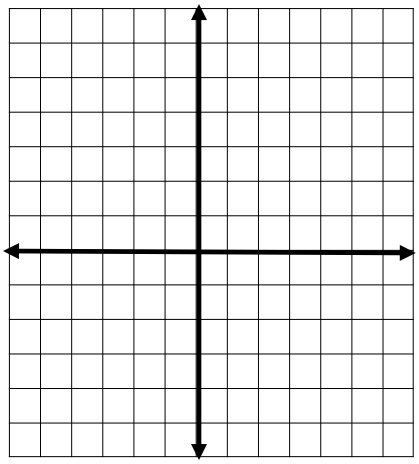
11. $4x^3 - 7x^2 - 5x + 6 = 0$ 12. $x^4 - 6x^3 + 14x^2 - 24x + 40 = 0$ 13. $3x^3 - 10x^2 + 31x + 26 = 0$

14. $2, 1 + i, 1 - i$ 15. $-1 - 3i, -1 + 3i, 2, -1$

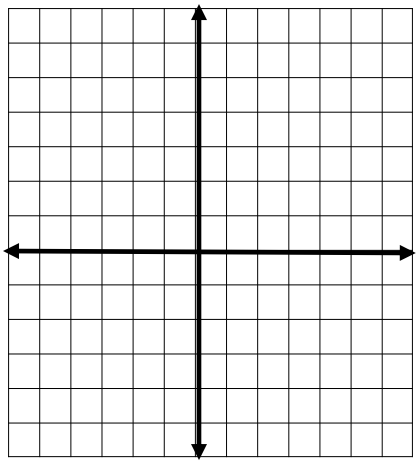
1. To graph a polynomial function:
    a. Find the zeros of the function. Remember zeros = x-intercepts so graph these points on the x-axis.
    b. Find the y-intercept
    c. Determine the end behavior of the function based on the degree and the leading coefficient.
    d. Using the end behavior and the intercepts to make a smooth curve.

    Examples: Graph each function.

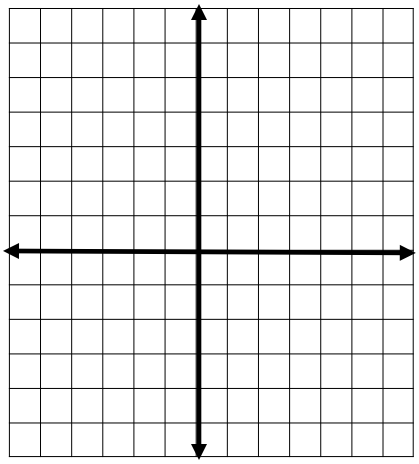
1. $y = -2(x^2 - 9)(x + 4)$



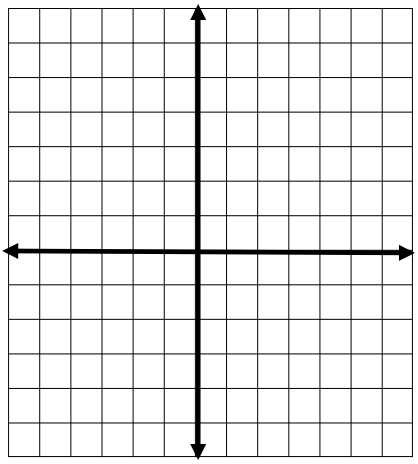
2. $y = (x^2 - 4)(x + 3)$



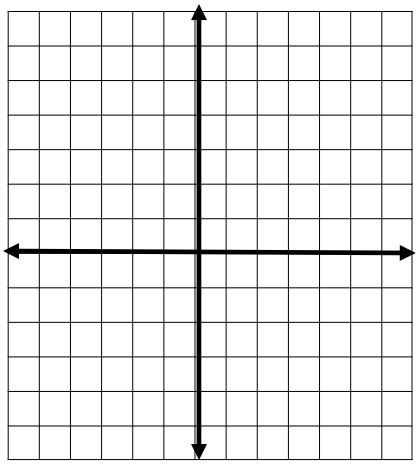
3. $y = -1(x^2 - 25)(x^2 - 4)$



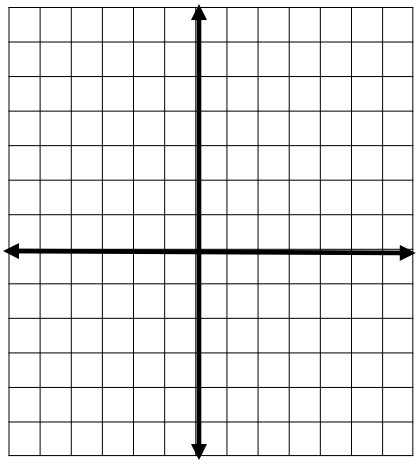
4. $y = 1/4(x + 2)(x - 1)^2$



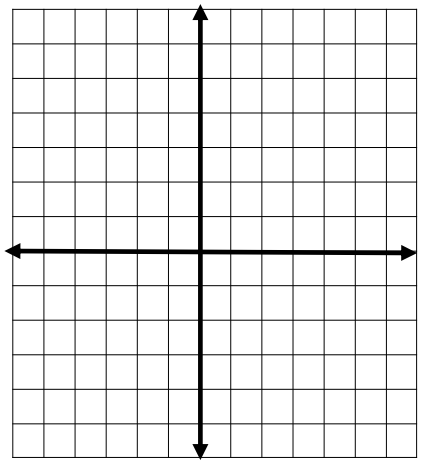
5. $y = 1/5(x - 3)^2(x + 1)^2$



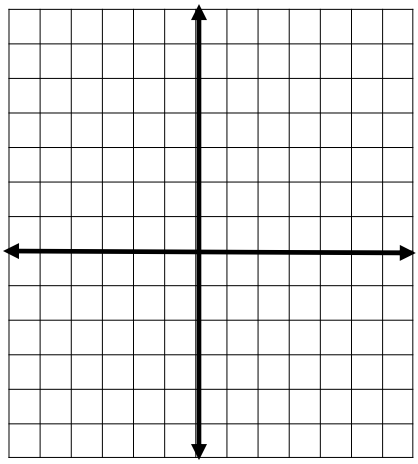
6. $y = (x + 1)^3(x - 4)$



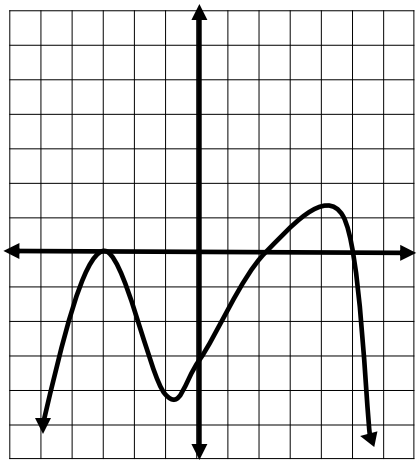
7. $y = x(x - 1)(x + 5)$

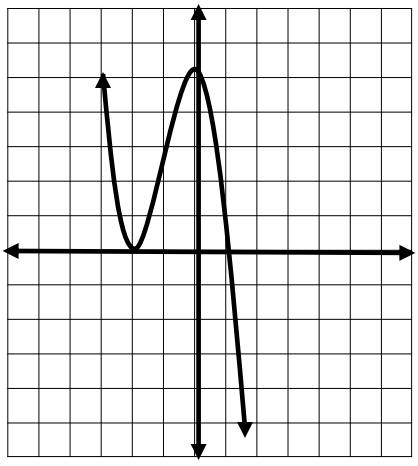


8. $y = x^2(x + 4)(x - 3)$



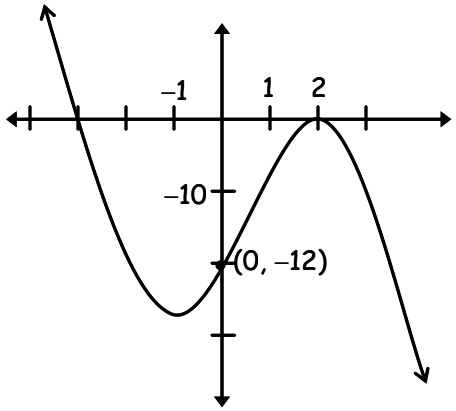
Write the equation of the graph below in standard form. Be sure to find "a".



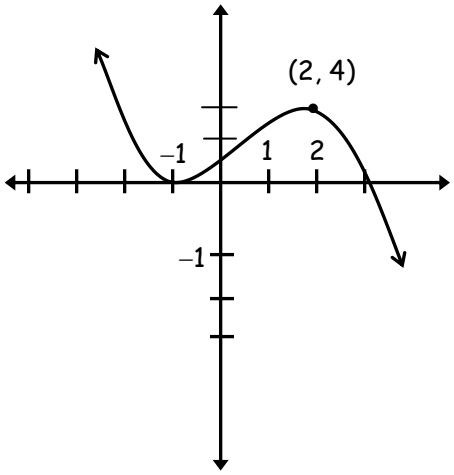


FCC3: day 9 Find the equation of the function shown:
Leave the answer in factored form. (Make sure to find the " a " value.)

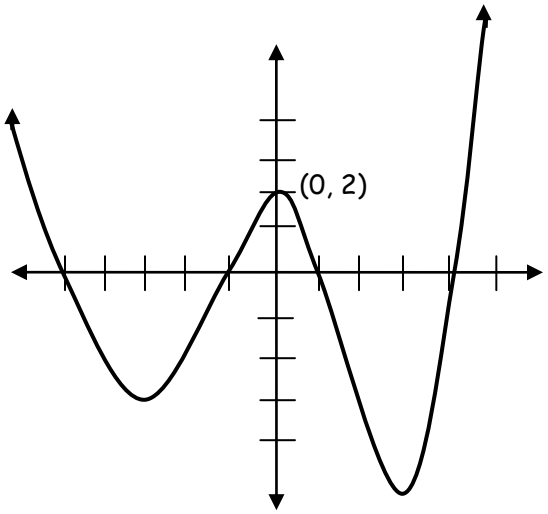
1. _____



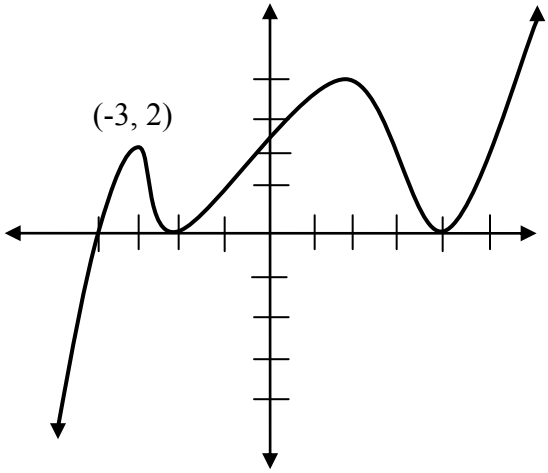

2. _____




3. _____




4. _____




5. Write the equation of the polynomial with zeros at 3, -4, and 1 through the point (-1,-6).

6. Write the equation of the polynomial with zeros at -3, -1(mult of 2), and 2 through the point (0, -12).

7. Write the equation of the polynomial with zeros at 1 (mult of 2), -5, and -3(mult of 2) through the point (-1,-5)

Answers: 1. y = -1(x + 3)(x - 2)(x - 2)
2. y = -4/9(x + 1)(x + 1)(x - 3)
3. y = 1/10(x + 5)(x - 1)(x + 1)(x - 4)
4. y = 2/49(x + 4)(x + 2)(x + 2)
5. y = -1/4(x - 3)(x + 4)(x - 1)
6. y = 2(x + 3)(x + 1)( x + 1)(x - 2)
7. y = -5/64(x - 1)(x - 1)(x + 5)(x + 3)(x + 3)

FCC# Unit 4 Polynomial Functions Review#1

1. Find a polynomial equation having roots –2 and $3 + i$.

2. Divide $x^4 - 3x^3 + 18x^2 - 12x + 16$ by $x - 3$ using long division.

3. Find all zeros for $p(x) = 2x^4 + 3x^3 + 6x^2 + 12x - 8$ if $2i$ is a zero.

4. One root of $2x^3 - 10x^2 + 9x - 4 = 0$ is 4. Find the other roots.

5. If $3 + 2i$ is a zero of a polynomial, what has to be another zero?

6. Describe the end behavior of each: (a) $f(x) = x^5 - x^3 - x^2 + x + 2$; (b) $h(x) = -x^4 - 9x^2$

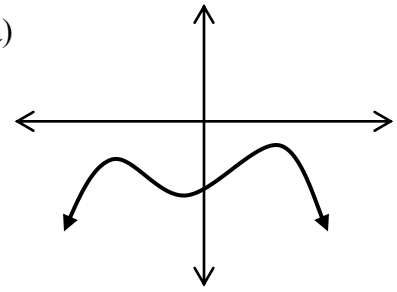
7. Approximate to the nearest tenth the real zeros of $f(x) = x^3 - 6x^2 + 8x - 2$. (Use a calculator)

8. For $y = x(x + 3)(x - 1)^2$, determine the zeros and their multiplicity.

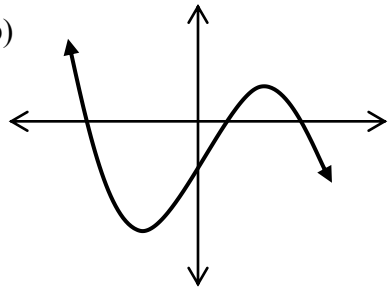
9. Write a polynomial function with zeros 1 and 2 (of multiplicity 3) in factored form.

10. Determine if the degree of the functions below is even or odd. How many real zeros does each have?

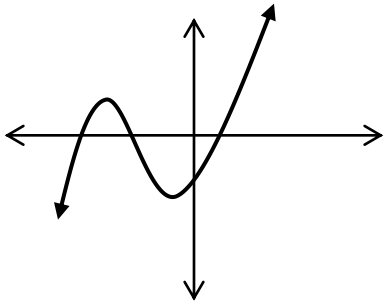
a)



b)



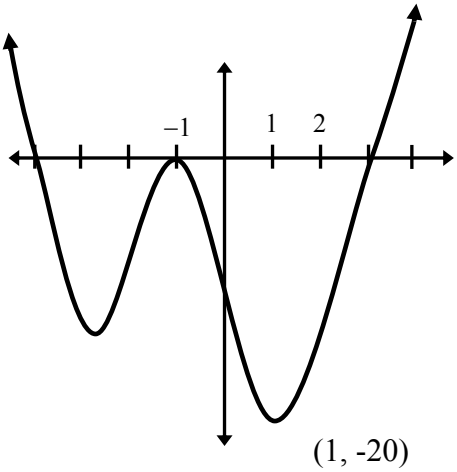
c)



11. Use synthetic division to find $f(-2)$ if $f(x) = 4x^5 + 10x^4 - 11x^3 - 22x^2 + 20x + 10$.

12. Factor: $2x^3 + 15x^2 - 14x - 48$ if $(x - 2)$ is a factor.

13. Write the following equation of the graph.




Put in factored form. _____

Sketch each of the following graphs:

14. $f(x) = (x-4)^3$

15, $f(x) = -2x^3 + 4$

16. $y = -2(x+3)(x-3)(x+4)$

17. $Y = (x-3)^2 (x+1)^2$

KEY:

1) $x^3 - 4x^2 - 2x + 20 = 0$ 2) $x^3 - 5x^2 + 3x - 3$ R 7 3) $\{ \frac{1}{2}, -2, \pm 2i \}$ 4) $\{ \frac{1}{2} \pm \frac{1}{2}i \}$ 5) $3 - 2i$

6) (a) low to high (b) low to low 7) 0.3, 1.5, 4.2 8) {0, –3, 1 (DR) } 9) $y = (x - 1)(x - 2)^3$

10) (a) even, none (b) odd, 3 (c) odd, 3 11) $f(-2) = 2$ 12) $(x - 2)(2x + 3)(x + 8)$

13.) $y = (-1/2)(x + 4)(x + 1)^2(x - 3)$

Graphs 14-17 done in class

Review #2 for test on Polynomials Name_____

I. Perform the indicated operations and put all final answers in simplest form.

1. $\left(\frac{-2}{3}\right)^{-2} - \left(\frac{1}{4}\right)^{0}$

2. $\frac{\left(-3x^4y^{-1}\right)^{-2}}{4^{-1}x^{-5}y^{3}}$

3. $\frac{4^{2k+3}}{4^{3-2k}}$

4. $\left(2^{x+3}\right)^{2} \cdot 2^{3x-1}$

5. Solve for x: $\left(b^2 \cdot b^x\right)^3 = \frac{b^x}{b^2}$

II. 6. Express answers in scientific notation: a) $\frac{3 \times 10^{-6}}{\left(2 \times 10^{5}\right)\left(5 \times 10^{-3}\right)}$

b) $(4 \times 10^{3})(3.2 \times 10^{-7})$

III. Multiply.

7. $(3x-2)^3$

8. $(a+5b)^5$

IV. Use Synthetic Division:

9. $\left(2x^4 - 5x^2 - 20\right) \div (x+2)$

Use Long Division:

10. $\left(8x^3 - 30x + 10\right)(2x-3)^{-1}$

11. $(28x^5y^3 - 32x^2y^8 - 4x^2y^2) \div (-4x^2y^2)$

12. Use synthetic division: $(6x^3 - 28x^2 + 19x + 3)(3x - 2)^{-1}$

V. Factor the following completely!

13. $4y^{2a} + 11y^a - 3$

14. $y^{3n} - 1$

15. $16x^3 + 2$

16. $x^{2n} - 49$

17. $n^4 - 5n^2 + 4$

18. $n^2m + 3n^2 - 9m - 27$

19. $216a^3 - 125b^6$

20. $6y^4 - 2y^2 - 4$

21. $a^{6n} - 2a^{3n} - 15$

22. $2a^{4k} - 162$

23. $54 - 9k - 3k^2$

24. $8y^4 + 50x^4$

25. $12x^2 - 68x + 40$

26. $90n^2 - 160$

27. $16 - 10c + c^2$

28. $64 + x^6$

29. $-x^2 + 2x - 1$

30. $15x^2 - 14xy - 8y^2$